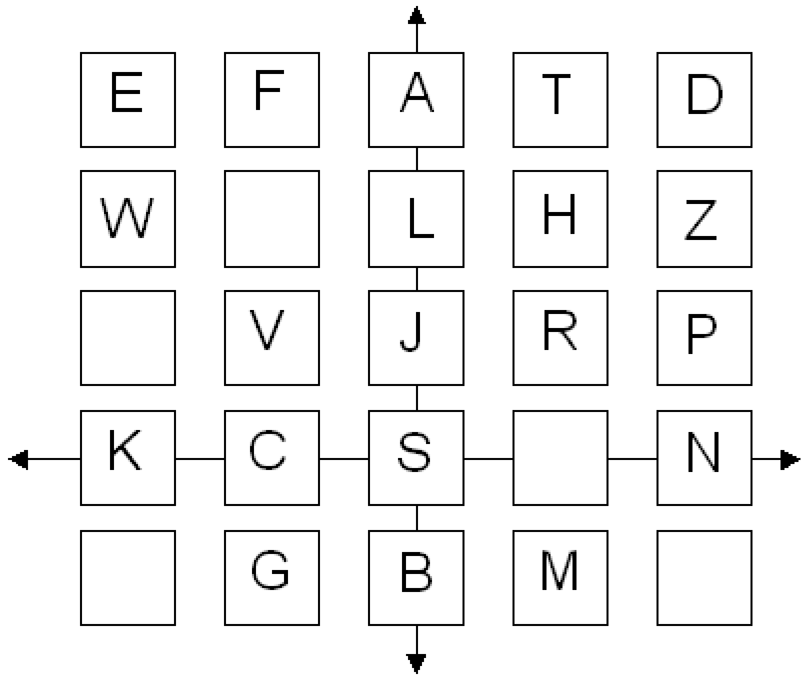
Algebra II **Class, Take Your Seats** *Answer Key*



CLUE Worksheet

CLUE	NAME	Possible Ordered Pairs
1		
2	Aristotle	(0, 3)
3	Bernoulli	(0, 1), (0, -1), (2, 0), (-2, 0)
4	Cauchy	(1, 1), (-1, 0)
5	Diophantus	(2, 3)
6	Euclid	(2, 1), (-2, 3)
7	Fibonacci	(-1, 3)
8	Galois	(-1, -1)
9	Hilbert	(2, 0), (1, 2)
10	Jacobi	(0, 1)
11	Kepler	(-2, 0)
12	Lagrange	(1, 2), (0, -1), (-1, 1), (0, 2), (1, -1), (2, 1), (2, 0), (-1 ,0)
13	Mobius	(0, 1), (-1, 3), (1, -1)
14	Napier	(2, 0)
15	Pythagoras	(0, 3,), (1, 2), (2, 1)
16	Riemann	(1, 1)
17	Saccheri	(0, 0), (1, 2)
18	Taylor	(1, 3)
19	Venn	(0, 0), (-1, -1), (-1, 1), (-2, 0)
20	Weil	(-2, 2), (-2, 0)
21	Zeno	(2, 2)

Solutions to Problems:

CLUES:

2. Aristotle is seated on the circle $x^2+y^2=9$.

   The center of the circle is (0, 0) and the radius is 3.
   The only point with integral coordinates in our domain is (0, 3).

3. Bernoulli is seated on the ellipse $x^2+4y^2=4$.

   The ellipse can be rewritten as $\frac{x^2}{4}+\frac{y^2}{1}=1$, where the center is (0, 0) and a =2 and b=1. The four vertices are (0, 1), (0, -1), (2, 0), (-2, 0).

4. Cauchy sits on the line 2y – 1 = x.

   In slope-intercept form, we get $y=\frac{1}{2}x+\frac{1}{2}$, which passes through (1, 1) and (-1, 0).

5. Diophantus is located at one of the foci of the hyperbola $\frac{(y+2)^2}{16}-\frac{(x-2)^2}{9}=1$.

   The center is at (2, -2), a = 3, and b = 4. Therefore, c = 5, so the foci are at (2, 3) and (2, -7), but the latter is not in our domain.

6. (0,7) and (4,5) are two consecutive vertices of a square. Euclid sits at one of the other vertices of the square.

   There are two ways in which the square can be generated, but only one way yields vertices in our domain; (2, 1), (-2, 3).

7. Fibonacci sits on the parabola $y=-x^2+2x+6$.

   The parabola can be rewritten as $y-7=-(x-1)^2$, where the vertex is (1, 7).
   It opens down and passes through (3, 3) and (-1, 3).

8. Galois sits at the intersection of $y=-x^2$ and $y=-x-2$.
   Solving simultaneously, we get $-x^2=-x-2$ or $x^2-x-2=0$. So, x = -1, 2.
   The two points of intersection are (-1, -1) and (2, -4).

9. Hilbert is seated on the parabola $y=x^2-5x+6$.

   The parabola can be rewritten as $y+\frac{1}{4}=-\left(x-\frac{5}{2}\right)^2$, where the vertex is $\left(\frac{5}{2},-\frac{1}{4}\right)$.
   The parabola passes through (2, 0), (3, 0), and (1, 2).

10. Jacobi is located at the center of the hyperbola $4x^2 - 9y^2 + 18y + 27 = 0$.

The hyperbola can be rewritten as $\frac{(y-1)^2}{4} - \frac{x^2}{9} = 1$.
So, the center is at (0, 1).

11. Kepler sits at one of the vertices of the hyperbola $\frac{(x-1)^2}{9} - \frac{y^2}{25} = 1$.

The center of the hyperbola is (1, 0). The hyperbola opens left and right, and a = 3 and b = 5, so the vertices are located at (4, 0) and (-2, 0).

12. Lagrange sits on the circle $x^2 + y^2 - x - y - 2 = 0$.

The circle can be rewritten as $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{5}{2}$.

The center of the circle is at $\left(\frac{1}{2}, \frac{1}{2}\right)$. The circle passes through the points (1, 2), (0, -1), (-1, 1), (0, 2), (1, -1), (2, 1), (2, 0), and (-1, 0).

13. Mobius sits on the line $y = -2x + 1$.

The line has a slope of –2 and a y-intercept of 1. So, it passes through the points (-1, 3), (0, 1), and (1, -1).

14. Napier is seated at the center of the circle $(x-2)^2 + y^2 = 49$.

The center of the circle is at (2, 0).

15. Pythagoras is located on the hypotenuse of the right triangle, whose vertices are (-1,4), (3,0), and (-1,0).

It is an isosceles right triangle and the slope of the hypotenuse is –1. So, the hypotenuse passes through the points (0, 3), (1, 2), and (2, 1).

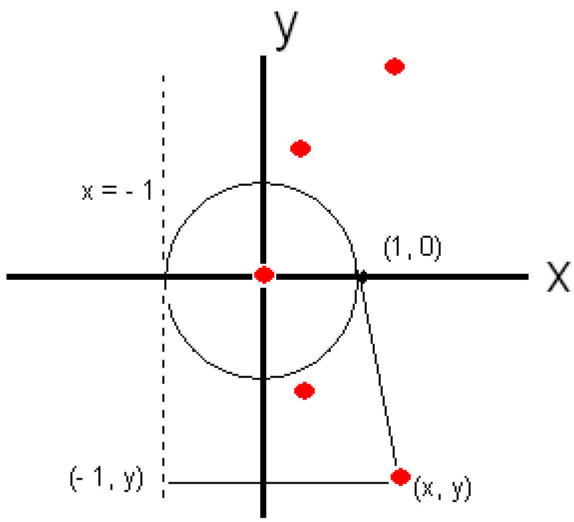
16. Riemann sits at the focus of the parabola $(y-1)^2 = 12(x+2)$.

Since 4p = 12, p = 3, the distance from the focus to the vertex.
The vertex of the parabola is located at (-2, 1), so the focus is at (1, 1).

17. A variable circle is always tangent to x = -1 and passes through (1,0). Saccheri sits on the locus of the center of that circle.

The locus of the center of the circle that is always tangent to the line x= -1 and passes though the point (1, 0) is a parabola whose center is the origin and which opens to the right.

Let (x, y) be the center of such a circle. Then the distance from (x, y) to (-1, y) must be equal to the distance from (x, y) to (1, 0) since they represent radii of the circle.



So, set the two distances equal and solve for the parabola:

$$\sqrt{(x+1)^2+(y-y)^2}=\sqrt{(x-1)^2+(y-0)^2}$$

$$x^2+2x+1=x^2-2x+1+y^2$$

$$y^2=4x$$

Therefore, some of the points with integral values that satisfy this parabola are: (0, 0), (1, 2), and (1, -2).

18. Taylor sits at the center of the ellipse $5x^2-10x+9y^2-54y+41=0$.

The ellipse can be rewritten as $\frac{(x-1)^2}{9}+\frac{(y-3)^2}{5}=1$, so the center is (1, 3).

19. Venn is located on the circle $x^2+2x+y^2=0$.

The circle can be rewritten as $(x+1)^2+y^2=1$. The center is (-1, 0) and the radius is 1. So, the circle passes through (-1, 1), (-1, -1), (-2, 0), and (0, 0).

20. Weil sits at one of the endpoints of the minor axis of the ellipse $\frac{(x+2)^2}{16}+\frac{(y-1)^2}{1}=1$.

The center of the ellipse is (-2, 1) and the length of the minor axis is 2, so the end points of the minor axis are (-2, 2) and (-2, 0).

21. Zeno sits at one of the foci of the ellipse $\frac{(x-2)^2}{4}+\frac{y^2}{8}=1$.

The center is (2, 0) and a = $2\sqrt{2}$, b = 2, so c = 2. Therefore, the foci are located at (2, 2) and (2, -2).