Methods of Integration

If you cannot simply integrate using the basic formulas above, try one of the following methods:

- **Integration by substitution**: a method by which some variable is substituted for part of the function f(x) (generally some complicated function within the larger function) in order to make integration simpler.

- **Integration by parts**: a method of integrating two functions multiplied together (the opposite of the product rule for derivatives), following the formula:

$$\int u\,dv = uv - \int v\,du$$

- **Integration by Trig Substitution**:

  If the integral contains trig expressions, try substituting some of the basic trig identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2A = 2\sin A\cos A \qquad \cos 2A = \cos^2 A - \sin^2 A$$

  If the integral contains the sum or difference of two squares, set up right triangles and make appropriate trig substitutions.

- **Numerical Integration**: When symbolic methods fail, use of some numerical approximation method will give useful answers along a specified interval. Most calculators enact these methods to give extremely exact answers by using very tiny subdivisions.

    - **RAM**: Rectangular Approximation Method.
      $\int \cong \Sigma_i \, y_i(\delta x)$ where y_i represents the heights of the successive rectangles and δx is the common width of the rectangles (the division).

    - **TRAPPROX**: Trapezoidal approximation.
      $\int \cong \frac{1}{2}(\delta x)[y_0 + 2(y_1 + ... + y_{n-1}) + y_n]$

    - **Simpson's rule:** Fitting parabolic segments beneath a curve.
      $\int \cong (b-a)[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + ... + 2y_{n-2} + 4y_{n-1} + y_n]/3n$
      where the interval is between *a* and *b* and *n* is an (even) number of subdivisions. (With higher number of subdivisions, smaller individual divisions are achieved.)