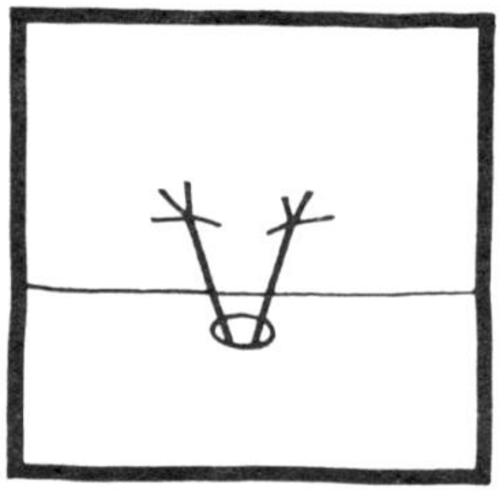
Droodle for Calculus A.P. Exam

A Puzzle by David Pleacher

Can you name this droodle?



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title." The droodle above was drawn by Roger Price and published in his book called, *Droodles*.

To determine the titles to this droodle, you must first solve the 35 problems in the puzzle and find the corresponding answers. Then replace each numbered blank in the puzzle with the letter corresponding to the answer for that problem and that will give you the titles.

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Title 1:

__     __ __ __ __ __ __      __ __ __ __     __ __ __     __ __
26     33  5 34  2 13 29       7 22 28  2     28  7  5     28 35

__ __ __ __  .
12 13 28 12
```

Title 2:

__ __ __   __ __ __ __   __ __ __ __ __   __ __   __
28 7 5   34 26 30 20   28 29 13 13 12   5 11   26

__ __ __ __ __ __   __ __ __ __ __ __.
1 13 12 13 29 28   22 12 30 26 11 1

Title 3:

__ __   __ __ __ __ __   __ __ __ __   __ __ __
3 15   4 3 17 10 23   18 8 17 31   32 9 16

__ __ __ __ __ __   __   __ __ __ __   __ __ __ __ __ __
24 3 25 6 9 21   3   27 4 17 23   19 21 17 16 15 6

__ __ __ __ .
32 16 17 14

Here are the choices for your answers:

A. -52

B. -24

C. -12

D. -32

E. -1

F. 0

G. 0.277

H. $-2 \le x \le 2$

I. 1

J. $-2 < x < 2$

K. $\frac{\pi}{6}$

L. 1.090

M. $\frac{1}{e-1}$

N. $\frac{\pi}{3}$

O. $\frac{5}{2}$

P. 2.887

R. 3

S. $\frac{7}{2}$

T. $\pm\sqrt{\frac{27}{2}}$

V. 6

W. 8

a. 10

b. 12

c. 16

Answer choices (continued):

d. $\ln(sec(1))$

e. $\dfrac{(x^3+7)(5x^4-1)-(x^5-x+2)(3x^2)}{(x^3+7)^2}$

f. limit does not exist

g. $\dfrac{2\sqrt{3}}{5}x^{\frac{5}{2}}+C$

h. 2x cos(2x) - $2x^2$ sin(2x)

i. $\int_{-2}^{4}(8+2x-x^2)dx$

j. $\dfrac{5x^4-1}{3x^2}$

k. $\int_{-2}^{4}(x^2-2x-8)dx$

l. y - 45 = 66 (x - 3)

m. $-\dfrac{(3x^2y+y^3)}{(3xy^2+x^3)}$

n. $\dfrac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$

o. $\dfrac{7}{6}e^{3x^2}+C$

p. $\pi(3^{\pi x-1})$

r. 9x - 8y - 26 = 0

s. $\pi\ln3(3^{\pi x})$

t. -4 sin 2x

u. $\pi\int_{0}^{\frac{\pi}{2}}(1+\sin^2 x)^2\,dx$

v. $\dfrac{-10}{3}(5-x)^{\frac{3}{2}}+\dfrac{2}{5}(5-x)^{\frac{5}{2}}+C$

w. $2x\sin^2(x^2)$

x. y + 45 = 66 (x - 3)

y. $\dfrac{1}{4}e^{4x}+C$

z. None of the above

A calculator may NOT be used on questions 1 – 28.

A graphing calculator is required for some questions 29 – 35.

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__ 1. If $g(x) = \frac{1}{32}x^4 - 5x^2$, determine g'(4).

__ 2. Determine the domain of the function $f(x) = \sqrt{4 - x^2}$.

__ 3. Determine $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$.

__ 4. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, determine $f'(x)$.

__ 5. Evaluate $\lim_{h \to 0} \frac{5\left(\frac{1}{2} + h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

__ 6. $\int x\sqrt{3x}\, dx =$

__ 7. Determine k so that $f(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & ; x \neq 4 \\ k & ; x = 4 \end{cases}$ is continuous for all x.

__ 8. Write the integral which correctly gives the area of the region consisting of all points above the x-axis and below the curve $y = 8 + 2x - x^2$.

__ 9. If $f(x) = x^2 \cos 2x$, determine $f'(x)$.

__10. Determine the equation of the line tangent to $y = 4x^3 - 7x^2$ at $x = 3$.

__ 11. $\int_{0}^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}} dx =$

__ 12. Determine a positive value c for x that satisfies the Mean Value Theorem for Derivatives for $f(x) = 3x^2 - 5x + 1$ on the interval [2, 5].

__ 13. Given $f(x) = 2x^2 - 7x - 10$, determine the absolute maximum of f(x) on [-1, 3].

__ 14. Determine $\frac{dy}{dx}$ if $x^3y + xy^3 = -10$.

__ 15. If $f(x) = \sqrt{1+\sqrt{x}}$, determine $f'(x)$.

__ 16. $\int 7xe^{3x^2} dx =$

__ 17. Determine the equation of the tangent line to $9x^2 + 16y^2 = 52$ through (2, -1).

__ 18. A particle's position is given by $s = t^3 - 6t^2 + 9t$. What is the acceleration at time $t = 4$?

__ 19. If $f(x) = 3^{\pi x}$, then $f'(x) =$

__ 20. Determine the average value of $f(x) = \frac{1}{x}$ from $x = 1$ to $x = e$.

__ 21. If $f(x) = \sin^2 x$, determine $f'''(x)$.

__ 22. Determine the slope of the tangent line to $y = x + \cos xy$ at (0, 1).

__ 23. $\int e^{x}\left(e^{3x}\right)dx =$

__ 24. $\lim_{x\to 0} \frac{2\tan^{3}(2x)}{x^{3}} =$

__ 25. A solid is generated when the region in the first quadrant bounded by the graph of y = 1 + $\sin^2$x, the line $x = \frac{\pi}{2}$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluation what integral?

__ 26. If $y = \left(\frac{x^{3}-2}{2x^{5}-1}\right)^{4}$, determine $\frac{dy}{dx}$ at $x = 1$.

__ 27. $\int x\sqrt{5-x}\,dx =$

__ 28. If $\frac{dy}{dx} = \frac{x^{3}+1}{y}$, and y = 2 when x = 1, then, when x = 2, y =

Graphing calculators may now be used for #29 – 35.

__ 29. The graph of $y = 5x^4 - x^5$ has an inflection point at x =

__ 30. The average value of $f(x) = e^{4x^2}$ on the interval $\left[-\frac{1}{4}, \frac{1}{4}\right]$ is

__ 31. $\int_0^1 \tan x\, dx =$

__ 32. $\frac{d}{dx}\int_0^{x^2} \sin^2 t\, dt =$

__ 33. Approximate $\int_0^1 \sin^2 x\, dx$ using the Trapezoidal Rule with n = 4, to three decimal places.

__ 34. A 20-foot ladder slides down a wall at 5 ft/sec. At what speed is the bottom sliding out when the top is 10 feet from the floor (in ft/sec)?

__ 35. If f(x) is continuous and differentiable and

$$f(x) = \begin{cases} ax^4 + 5x; & x \le 2 \\ bx^2 - 3x; & x > 2 \end{cases}$$ then $b =$