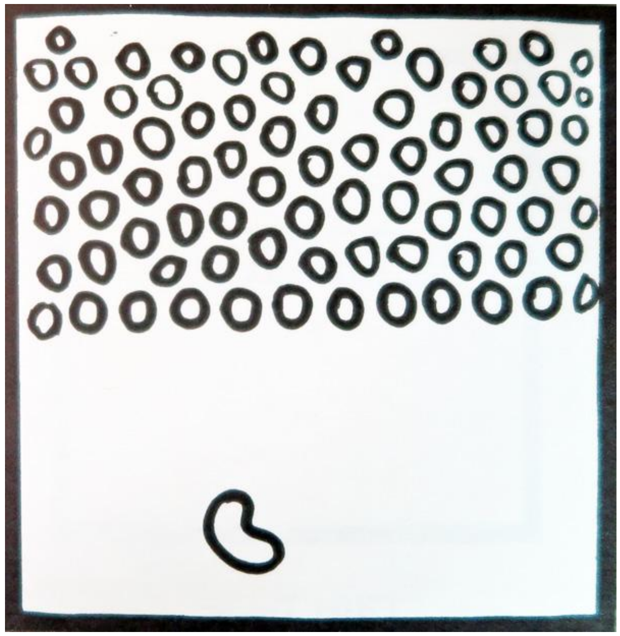
A Droodle for A.P. Calculus Exam

Puzzle and Answer Key by David Pleacher



"A Droodle is a borkley looking sort of drawing that doesn't make any sense until you know the correct title." – Roger Price

Caption for the picture:

"P A T R I O T I C   B E A N   T R Y I N G
6 4 11 8 12 3 11 12 16   13 9 4 2   11 8 7 12 2 5

T O   J O I N   T H E   P E A S   C O R P S."
11 3   1 3 12 2   11 10 9   6 9 4 14   16 3 8 6 14

J 1. If $f(x)=x^{\frac{3}{2}}$, then $f'(4)=3$

$$f'(x)=\frac{3}{2}x^{\frac{1}{2}}=\frac{3}{2}\sqrt{x}$$

$$f'(4)=\frac{3}{2}\sqrt{4}=3$$

N 2. $\lim\limits_{n\to 0}\frac{7n^3-5n}{n^3-2n^2+1}=7$

$$\lim_{n\to 0}\frac{7n^3-5n}{n^3-2n^2+1}=\lim_{n\to 0}\frac{\frac{7n^3}{n^3}-\frac{5n}{n^3}}{\frac{n^3}{n^3}-\frac{2n^2}{n^3}+\frac{1}{n^3}}=\lim_{n\to 0}\frac{\frac{7}{1}-\frac{5}{n^2}}{\frac{1}{1}-\frac{2}{n}+\frac{1}{n^2}}=\frac{7-0}{1-0+0}=7$$

O 3. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=-\frac{\left(x^2+y\right)}{\left(x+2y^2\right)}$

$$\frac{d}{dx}\left(x^3+3xy+2y^3\right)=\frac{d}{dx}(17)$$

$$3x^2+3x\frac{dy}{dx}+y\frac{d}{dx}(3x)+6y^2\frac{dy}{dx}=0$$

$$3x\frac{dy}{dx}+6y^2\frac{dy}{dx}=-3x^2-3y$$

$$\frac{dy}{dx}=\frac{-3\left(x^2+y\right)}{3\left(x+2y^2\right)}=-\frac{\left(x^2+y\right)}{\left(x+2y^2\right)}$$

A 4. If the function *f* is continuous for all real numbers and if $f(x)=\frac{x^2-4}{x+2}$ when $x\neq 2$,

Then $f(-2)=$ -4

$$\lim_{n\to -2}\frac{x^2-4}{x+2}=\lim_{n\to -2}\frac{(x-2)(x+2)}{x+2}=\lim_{n\to -2}(x-2)=-4$$

G 5. The area of the region enclosed by the curve $y=\frac{1}{x-1}$, the x-axis, and the lines x = 3 and x = 4 is $\ln\frac{3}{2}$

$$A=\int_3^4\frac{1}{x-1}dx=\ln|x-1|\Big|_3^4=\ln 3-\ln 2=\ln\frac{3}{2}$$

P 6. An equation of the line tangent to the graph of $y=\frac{2x+3}{3x-2}$ at the point (1, 5) is 13x + y = 18

$$\frac{dy}{dx}=\frac{(3x-2)\frac{d}{dx}(2x+3)-(2x+3)\frac{d}{dx}(3x-2)}{(3x-2)^2}=\frac{-13}{(3x-2)^2}$$

$$m=\frac{-13}{(3\bullet 1-2)^2}=-13$$

$$y-5=-13(x-1) \qquad y=-13x+18$$

Y 7. If $y=\tan x-\cot x$, then $\frac{dy}{dx}=\sec^2 x-\left(-\csc^2 x\right)=\sec^2 x+\csc^2 x$

R 8. If h is the function given by $h(x)=f\left(g(x)\right)$, where $f(x)=3x^2-1$ and $g(x)=|x|$,

then $h(x)=f\left(g(x)\right)=3\left(|x|\right)^2-1=3x^2-1$

E 9. If $f(x)=(x-1)^2\sin x$, then $f'(0)=1$

$$f'(x)=(x-1)^2\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}\left((x-1)^2\right)=(x-1)^2\cos x+\sin x\left(2(x-1)\right)$$

$$f'(0)=1\bullet 1+0=1$$

H 10. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=6t-2$. If the velocity is 25 when t = 3 and the position is 10 when t = 1, Then the position $x(t)=t^3-t^2+4t+6$

$$v(t)=\int a(t)\,dt=\int(6t-2)\,dt=3t^2-2t+k$$

$$v(3)=25=3\left(3^2\right)-2(3)+k$$

$$k=4 \qquad \text{so} \quad v(t)=3t^2-2t+4$$

$$x(t)=\int v(t)\,dt=\int\left(3t^2-2t+4\right)dt=t^3-t^2+4t+C$$

$$x(1)=10=1-1+4+C$$

$$C=6 \qquad \text{so} \quad x(t)=t^3-t^2+4t+6$$

_T_ 11. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx = 2\sqrt{x^3+1}+C$

Let $u = x^3+1$

Then $du = 3x^2dx$

$$\int \frac{3x^2}{\sqrt{x^3+1}}\,dx = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}du = 2(u)^{\frac{1}{2}}+C$$

$$= 2\sqrt{x^3+1}+C$$

_I_ 12. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

$$f'(x) = (x-2)\frac{d}{dx}(x-3)^2 + (x-3)^2\frac{d}{dx}(x-2) = (x-3)(3x-7)$$

Set $f'(x) = 0$ to find possible relative max/min

$x = 3, \ \frac{7}{3}$

Find $f''(x)$ to check concavity:

$f''(x) = 6x-16$

$f''(3) = 32$ so it is concave up and a rel minimum

$f''\left(\frac{7}{3}\right) = -2$ so it is concave down and a rel maximum

_B_ 13. The slope of the <u>normal</u> to the graph of $y = 2\ln(\sec x)$ at $x = \frac{\pi}{4}$ is

$$y' = 2\,\frac{\frac{d}{dx}(\sec x)}{\sec x} = 2\,\frac{\sec x \tan x}{\sec x} = 2\tan x$$

$$m_{\text{tangent}} = 2\tan\left(\frac{\pi}{4}\right) = 2$$

$$m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = -\frac{1}{2}$$

S 14. $\int \left(x^2+1\right)^2 dx =$

$$\int \left(x^2+1\right)^2 dx = \int \left(x^4+2x^2+1\right) dx =$$

$$= \frac{1}{5}x^5 + 2 \bullet \frac{1}{3}x^3 + x + K$$

$$= \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + K$$

X 15. $\frac{d}{dx}\int_0^x \cos\left(2\pi u\right) du = \cos(2\pi)$ by the Fundamental Theorem of Calculus

C 16. What is the minimum value of $f\left(x\right) = x\ln x$?

$$f\left(x\right) = x\ln x$$

$$f'\left(x\right) = x\frac{d}{dx}\left(\ln x\right) + \ln x\frac{d}{dx}\left(x\right)$$

$$= x \bullet \frac{1}{x} + \ln x = 1 + \ln x$$

Set $1+\ln x = 0$ to solve for possible relative max/min points

$\ln x = -1$

So, $x = e^{-1}$

$$y = x\ln x = e^{-1}\ln\left(e^{-1}\right) = e^{-1}\left(-1\right) = -\frac{1}{e}$$

Now find the second derivative and set = 0 to check concavity:

$$f''(x) = \frac{1}{x}$$

$\frac{1}{e^{-1}} = \frac{1}{e} > 0$ so it is concave up and therefore a rel minimum

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The droodle used in this puzzle was drawn by Roger Price and appeared in his book called *Droodles.*

Answers: (units have been omitted)

A. -4

B. $-\frac{1}{2}$

C. $-\frac{1}{e}$

D. -3

E. 1

F. 0

G. $\ln\frac{3}{2}$

H. $t^3 - t^2 + 4t + 6$

I. $\frac{7}{3}$

J. 3

K. $3x^2|x| - 1$

L. $-\frac{x^2 + y}{x + 2y}$

M. nonexistent

N. 7

O. $-\frac{x^2 + y}{x + 2y^2}$

P. 13x + y = 18

Q. $\sqrt{x^3 + 1} + C$

R. $3x^2 - 1$

S. $\frac{x^5}{5} + \frac{2x^2}{3} + x + C$

T. $2\sqrt{x^3 + 1} + C$

U. $t^3 - t^2 + 9t - 20$

V. $\ln\frac{2}{3}$

W. $\frac{1}{2\pi}\cos(2\pi x)$

X. $\cos(2\pi x)$

Y. $\sec^2 x + \csc^2 x$

Z. None of the above