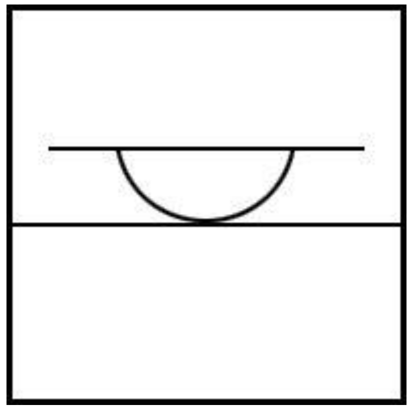
A Droodle for the A.P. Calculus Exam

A puzzle by David Pleacher



"A Droodle is a borkley looking sort of drawing that doesn't make any sense until you know the correct title." – Roger Price

Caption for the picture:

___	___	___	___	___	___	___	___ ,	___	___	___	___	___	___	___	___	___	___	___	___	___ .
15	14	5	13	8	13	11	11	9	6	4	4	3	9	5	8	13	8	16	10	4

To determine the title to this droodle, which was created by Roger Price and published in his book called *Droodles*, solve the 16 A.P. Calculus problems (from the 1993 AB Exam).
Then find the answers to each problem from the choices below.
Replace each numbered blank with the letter corresponding to the answer for that problem.
A calculator should not be used on the following problems except for #7.

___ 1. $\displaystyle\int_0^{\frac{1}{2}} \frac{8\,dt}{1+4t^2} =$

___ 2. At what value of x does the graph of $y = \dfrac{1}{x^2} - \dfrac{1}{x^3}$ have a point of inflection?

___ 3. An antiderivative for $\dfrac{1}{x^2-2x+2}$ is

___ 4. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

___ 5. If $f(x)=\left(x^2-2x-1\right)^{\frac{2}{3}}$, then $f'(0)$ is

___ 6. $\dfrac{d}{dx}\left(2^x\right)=$

___ 7. A particle moves along a line so that at time t, where $0\le t\le\pi$, its position is given by $s(t)=-4\cos t-\dfrac{t^2}{2}+10$. What is the velocity of the particle when its acceleration is zero?

___ 8. $\lim\limits_{\theta\to 0}\left(\dfrac{1-\cos\theta}{2\sin^2\theta}\right)=$

___ 9. The region enclosed by the x-axis, the line $x=3$, and the curve $y=\sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

___ 10. If $f(x)=e^{3\ln\left(x^2\right)}$, then $f'(x)=$

___ 11. $\displaystyle\int_0^{\sqrt{3}}\frac{dx}{\sqrt{4-x^2}}=$

___ 12. If $\dfrac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then, when $x = 2$, $y =$

___ 13. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of the distance between the bottom of the ladder and the wall (in feet per minute)?

___ 14. If the graph of $y = \dfrac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote of $x = -3$, then $a + c =$

___ 15. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

___ 16. The fundamental period of $2\cos(3x)$ is

The droodle used in this puzzle was drawn by Roger Price and appeared in his book called *Droodles.*

Answers: (units have been omitted)

A. $-\frac{4}{3}$

B. $-\frac{1}{3}$

C. 0

D. $\frac{1}{4}$

E. $\frac{7}{8}$

F. 1

G. $\frac{\pi}{3}$

H. 1.32

I. $\frac{4}{3}$

J. 2

K. 2.55

L. nonexistent

M. $2\sqrt{3}\pi$

N. 3

O. $\frac{2\pi}{3}$

P. π

Q. 2^{x-1}

R. 5

S. $\frac{9\pi}{2}$

T. $6(\ln x)e^{3\ln(x^2)}$

U. $2^x \ln 2$

V. $\ln(x^2 - 2x + 2)$

W. $6x^5$

X. $\frac{3}{x^2}e^{3\ln(x^2)}$

Y. $\operatorname{Arctan}(x-1)$

Z. None of the above