Age Equation Shirts – Solution Key

1. Determine $g'(-1)$ if $g(x)=5\left(4x-x^5\right)\left(x^2-3x\right)$

$$g'(x)=5\left(4x-x^5\right)\left(2x-3\right)+5\left(4-5x^4\right)\left(x^2-3x\right)$$
$$g'(-1)=5\left(-4+1\right)\left(-5\right)+5\left(-1\right)\left(4\right)$$
$$g'(-1)=75-20=55$$

2. Determine $f'(2)$ if $f(x)=5x^4\cos^2\left(3x-6\right)-25x^2$

$$f'(x)=\left(5x^4\right)\left(2\cos(3x-6)\right)\left(-\sin(3x-6)\right)+20x^3\cos^2(3x-6)-50x$$
$$f'(2)=0+(20)(8)(1)-100=60$$

3. Determine $y'(0)$ given $y=-13e^{\sin\left(4x^2-5x\right)}$

$$y'=-13e^{\sin\left(4x^2-5x\right)}\bullet\cos\left(4x^2-5x\right)\bullet\left(8x-5\right)$$
$$y'(0)=-13\bullet1\bullet(-5)=65$$

4. Determine the value of $-62\left(g'(3)\right)$ if $g(3)=9$ and $3x\,g(x)+6\sqrt{g(x)}=4\left(g(x)\right)^2+48x$

$$3x\,g'(x)+3g(x)+3\left(g(x)\right)^{-\frac{1}{2}}\bullet g'(x)=8g(x)g'(x)+48$$
$$9\,g'(3)+3g(3)+\frac{3}{\sqrt{g(3)}}\bullet g'(3)=8g(3)g'(3)+48$$
$$9\,g'(3)+27+g'(3)=72g'(3)+48$$
$$10g'(3)+27=72g'(3)+48$$
$$-21=62g'(3)$$
$$g'(3)=\frac{-21}{62}$$
$$\therefore -62\bullet g'(3)=21$$

5. Determine $h'(0)$, given $\mathrm{h}(x)=9\sin\left(4x\right)-3e^{3x}$

$$\mathrm{h}'(x)=9\cos\left(4x\right)\bullet4-3e^{3x}\bullet3$$
$$\mathrm{h}'(x)=36\cos\left(4x\right)-9e^{3x}$$
$$\mathrm{h}'(0)=36\cos\left(0\right)-9e^{0}$$
$$\mathrm{h}'(0)=36\bullet1-9\bullet1=27$$

6. Determine $\left.\frac{dy}{dx}\right|_{x=3}$ if $y = 42\ \ln\left(15x^2 - 3x\right) + 9x$

$$\frac{dy}{dx} = 42 \bullet \left(\frac{30x-3}{15x^2-3x}\right) + 9$$

$$\frac{dy}{dx} = 42 \bullet \left(\frac{10x-1}{5x^2-x}\right) + 9$$

$$\left.\frac{dy}{dx}\right|_{x=3} = 42 \bullet \left(\frac{29}{42}\right) + 9$$

$$\left.\frac{dy}{dx}\right|_{x=3} = 29 + 9 = 38$$

7. Given $y^4 - 5y = 700 - 350x$, Determine $\frac{dy}{dx}$ at the point $(2,0)$

$$4y^3\frac{dy}{dx} - 5\frac{dy}{dx} = -350$$

$$\frac{dy}{dx} = \frac{-350}{4y^3-5}$$

$$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{-350}{-5} = 70$$

8. Given $f(x) = 62x - \cos x + x\sin\left(x^3\right)$, Determine $f\,'(0)$

$$f\,'(x) = 62 + \sin x + x\cos\left(x^3\right) \bullet 3x^2 + \sin\left(x^3\right)$$

$$f\,'(0) = 62 + 0 + 0 + 0 = 62$$

9. Determine $k\,'(1)$ given $k(x) = 8x^5 + 3x\sin\left(2\pi\right) + \frac{6}{x^2} + 8e^3 - \frac{4}{x+1} + 15x^2$

$$k\,'(x) = 40x^4 + 0 - \frac{12}{x^3} + 0 - \frac{-4}{\left(x+1\right)^2} + 30x$$

$$k\,'(1) = 40 - 12 + 1 + 30$$

$$k\,'(1) = 59$$

10. If $h(t) = \pi^2 + t^3 + e^\pi$, Determine $\dfrac{dh}{dt}$ at $t = 4$

$$h'(t) = 0 + 3t^2 + 0$$
$$h'(4) = 48$$

11. If $s = -43 \ln \sqrt{t^2 + 2t - 1}$, Determine $\left.\dfrac{ds}{dt}\right|_{t=0}$

$$\frac{ds}{dt} = -43 \bullet \frac{1}{\sqrt{t^2 + 2t - 1}} \bullet \frac{d}{dt}\left(\sqrt{t^2 + 2t - 1}\right)$$
$$\frac{ds}{dt} = \frac{-43}{\sqrt{t^2 + 2t - 1}} \bullet \frac{1}{2}\left(t^2 + 2t - 1\right)^{\frac{-1}{2}} \bullet (2t + 2)$$
$$\frac{ds}{dt} = \frac{-43}{t^2 + 2t - 1} \bullet (t + 1)$$
$$\left.\frac{ds}{dt}\right|_{t=0} = \frac{-43}{-1} \bullet 1 = 43$$

<u>12 – 13</u> Use the following table to answer questions #12 and #13:

x	f(x)	f'(x)	g(x)	g'(x)
1	3	4	5	7
5	2	6	5	1

12. If $M(x) = f\left(g(x)\right)$, Then determine the value of $M'(1)$

$$M'(x) = f'\left(g(x)\right) \bullet g'(x)$$
$$M'(1) = f'\left(g(1)\right) \bullet g'(1)$$
$$M'(1) = f'(5) \bullet 7$$
$$M'(1) = 6 \bullet 7 = 42$$

13. If $H(x) = \left(g(x)\right)^3$, Then determine the value of $H'(5)$

$$H'(x) = 3\left(g(x)\right)^2 \bullet g'(x)$$
$$H'(5) = 3\left(g(5)\right)^2 \bullet g'(5)$$
$$H'(5) = 3(5)^2 \bullet 1$$
$$H'(5) = 75$$

BONUS:

$$\text{Age} = \left(\ln\left[\tan\frac{\pi}{4} + \int_0^7 e^x dx \right] \right)^2 + \sum_{n=1}^{\infty} \left[\ln\alpha \left| \frac{d}{dx} \log_a \sqrt{x} \right|_{x=1} \right]^n$$

$$\text{Age} = \left(\ln\left[1 + e^7 - 1 \right] \right)^2 + \sum_{n=1}^{\infty} \left[\ln\alpha \left| \frac{1}{2x\ln\alpha} \right|_{x=1} \right]^n$$

$$\text{Age} = \left(\ln\left[e^7 \right] \right)^2 + \sum_{n=1}^{\infty} \left[\ln\alpha \left| \frac{1}{2x\ln\alpha} \right| \right]^n$$

$$\text{Age} = (7)^2 + \sum_{n=1}^{\infty} \left[\frac{1}{2} \right]^n$$

$$\text{Age} = 49+1$$

$$\text{Age} = 50$$