Turvy with Applications of the Derivative -- Solution

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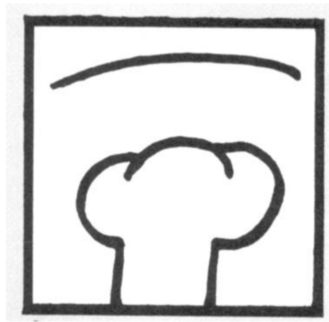
Correction to #7 by Samuel Iofel



Here is the title right-side-up: "Italian chef tossing pizza dough."

Here is the title upside-down: "Close-up of a Cabbage Patch Kid."

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1. Find the equation of the line normal to the curve $f(x) = x^3 - 3x^2$ at the point (1, -2).

   K. $3y - x = -7$

2. Find the equation of the line tangent to the curve $x^2 y - x = y^3 - 8$ at the point where x = 0.

   F. $12y + x = 24$

3. Determine the point(s) of inflection of $f(x) = x^3 - 5x^2 + 3x + 6$.

   Z. $\left(\frac{5}{3}, \frac{47}{27}\right)$

4. Determine the relative minimum point(s) of $f(x) = x^4 - 4x^3$.

   L. $(3, -27)$

5. A particle moves along a line according to the law $s = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

   Determine the total distance traveled between t = 0 and t = 4.

   H. 34

6. A particle moves along a line according to the law $s = t^4 - 4t^3$, where $t \geq 0$.
   Determine the total distance traveled between $t = 0$ and $t = 4$.
   G. 54

7. If one leg, AB, of a right triangle increases at the rate of 2 inches per second, while the other leg AC decreases at 3 inches per second, determine how fast the hypotenuse is changing (in **feet** per second) when AB = 6 feet and AC = 8 feet.
   N. $-\dfrac{1}{10}$

8. The diameter and height of a paper cup in the shape of a cone are both 4 inches, and water is leaking out at the rate of ½ cubic inch per second. Determine the rate (in inches per second) at which the water level is dropping when the diameter of the surface is 2 inches.
   S. $\dfrac{1}{2\pi}$

   The key is that the **diameter** is given to be 4 inches and not the radius.

Given $\dfrac{dV}{dt} = -\dfrac{1}{2}\dfrac{in^3}{\text{sec}}$ and $h = d = 4$ in.

Find $\dfrac{\text{dh}}{\text{dt}}$ when $d = 2$ which means $r = 1$ and therefore $h = 2$ since $r = \dfrac{1}{2}h$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$$

$$-\frac{1}{2} = \frac{\pi}{4}(2)^2\frac{dh}{dt}$$

$$-\frac{1}{2} = \frac{dh}{dt}$$

9. For what value of y is the tangent to the curve $y^2 - xy + 9 = 0$ vertical?
   E. ± 3

10. For what value of k is the line $y = 3x + k$ tangent to the curve $y = x^3$?
   T. ± 2

11. Determine the slopes of the two tangents that can be drawn from the point (3, 5) to the parabola $y = x^2$.
   U. 2 and 10

12. Determine the area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve $y = e^{-x^2}$.
   P. $\sqrt{\dfrac{2}{e}}$

13. A tangent drawn to the parabola $y = 4 - x^2$ at the point (1, 3) forms a right triangle with the coordinate axes. What is the area of this triangle?
   O. $\dfrac{25}{4}$

14. If the cylinder of largest possible volume is inscribed in a given sphere, determine the ratio of the volume of the sphere to that of the cylinder.
   D. $\sqrt{3}:1$

15. Determine the first quadrant point on the curve $y^2x = 18$ which is closest to the point (2, 0).
   B. $\left(3, \sqrt{6}\right)$

16. Two cars are traveling along perpendicular roads, car A at 40 mph, car B at 60 mph. At noon when car A reaches the intersection, car B is 90 miles away, and moving toward it. At 1PM, what is the rate, in miles per hour, at which the distance between the cars is changing?

    I. -4

17. A 26-foot ladder leans against a building so that its foot moves away from the building at the rate of 3 feet per second. When the foot of the ladder is 10 feet from the building, at what rate is the top moving down (in feet per second)?

    C. $\frac{5}{4}$ Note: When you solve the equation, you get $\frac{dy}{dt} = \frac{-5}{4}$ ft/sec

    where y represents the distance the top of the ladder moves down the wall.
    So, the rate at which the top of the ladder is moving down the wall is 5/4 ft/sec

18. A rectangle of perimeter 18 inches is rotated about one of its sides to generate a right circular cylinder. What is the area, in square inches, of the rectangle that generates the cylinder of largest volume?

    A. 18