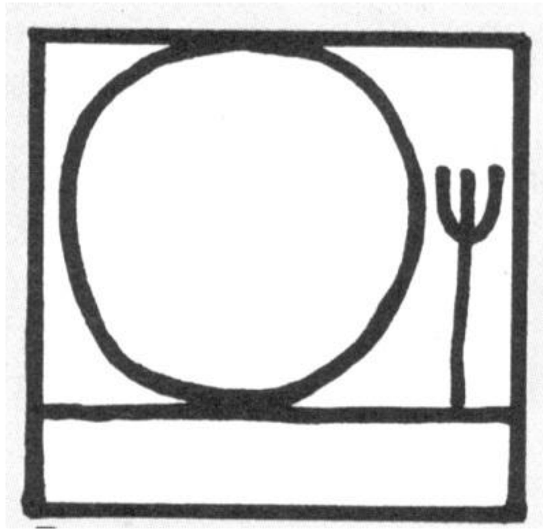
Turvy for Limits & Continuity-- A Puzzle by David Pleacher



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title."   In 1985, Games Magazine took the Droodle one step further and created the Turvy. Turvies have one explanation right-side-up and an entirely different one turned topsy-turvy. The Turvy above was created by David LaRochelle of Spring Lake Park, Minnesota and published in Games Magazine in May 1986.

Here is the title right-side-up:

___ ___ ___ ___ ___   ___ ___ ___   ___ ___ ___ ___   ___ ___   ___
6 10 12 16 17   14 3 13   19 12 10 14   8 2   21

___ ___ ___ ___ ___   ___ ___ ___ ___ ___ ___ ___ ___
15 16 21 13 11   20 13 19 1 8 21 17 17

Here is the title upside-down:

___ ___ ___ ___ ___ ___ ___   ___ ___ ___ ___ ___ ___
19 21 11 7 10 21 17   9 19 19 4 16 10

___ ___ ___ ___ ___ ___ ___ ___ ___ ___   ___ ___ ___ ___
15 14 21 6 3 21 11 16 13 15   5 14 19 7

___ ___ ___ ___   ___ ___ ___ ___ ___ ___
18 16 15 18   20 9 18 19 19 17

To determine the titles to this turvy, solve the 21 Limits and Continuity problems in this puzzle. Then replace each numbered blank in the puzzle with the letter corresponding to the answer for that problem.

___ 1. If $c \neq 0$, evaluate $\lim_{x \to c} \frac{x^3 - c^3}{x^6 - c^6}$

___ 2. $\lim_{x \to 0} (x-5)\cos(x) =$

___ 3. Evaluate $\lim_{x \to 0} \frac{2 - \sqrt{4-x}}{x}$

___ 4. Evaluate $\lim_{x \to \infty} \frac{5 - 6x}{2x + 13}$

___ 5. Evaluate $\lim_{x \to \infty} \frac{5x^2 - 6x + 9}{x^3 - 2x^2}$

___ 6. Determine the value of k that makes the function f(x) continuous on [0, 11], if f(x) is defined as follows:

$$f(x) = \begin{cases} k \bullet \sin \frac{(x+3)\pi}{6}, & x \leq 2 \\ \frac{3 - \sqrt{11-x}}{x-2}, & x > 2 \end{cases}$$

<u>7 – 8.</u>

Given $f(x) = \begin{cases} \ln x & \text{if } 0 < x < 1 \\ ax^2 + b & \text{if } 1 \leq x < \infty \end{cases}$

If $f(2) = 3$, determine the values of a and b for which $f(x)$ is continuous on the interval $(0, \infty)$.

___ 7. a =

___ 8. b =

___ 9. Evaluate $\lim_{x \to \infty} \frac{3x^3 + 9}{5x + 8}$

___ 10. Evaluate $\lim_{x \to -\infty} \frac{3x}{\sqrt{x^2 - 4}}$

___ 11. If $f(x) = \begin{cases} e^x & \text{for } 0 \leq x < 1 \\ x^2 e^x & \text{for } 1 \leq x < 5 \end{cases}$ determine $\lim_{x \to 1} f(x)$

___ 12. Evaluate $\lim_{x \to \infty} \frac{e^x}{1 - x^3}$

___ 13. Evaluate $\lim_{x\to 0}\frac{\sin 3x}{\sin 4x}$

___ 14. Given the function: $f(x)=\begin{cases}\frac{x^2-9}{x-3} & \text{for } x\neq 3\\ a & \text{for } x=3\end{cases}$

Determine the value of *a* which makes the function continuous.

<u>15 – 16.</u> Given the function: $f(x)=\begin{cases}\sin x & \text{if } x\le -\frac{\pi}{2}\\ a\sin x+b & \text{if } -\frac{\pi}{2}<x<\frac{\pi}{2}\\ 2\cos x & \text{if } x\ge \frac{\pi}{2}\end{cases}$

Determine the values of *a* and *b* so that the function *f*(x) is continuous for all values of x.

___ 15. a =

___ 16. b =

___ 17. Determine the points of discontinuity of the function $f(x)=\frac{1}{x^3-3x^2-x+3}$

___ 18. Given the function: $f(x)=\begin{cases}|3\text{-}x| & \text{if } x<7\\ ax-10 & \text{if } 7\le x<10\end{cases}$

Determine the value of *a* so that the function f(x) is continuous on the interval $(-\infty,10)$.

___ 19. Evaluate $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$

___ 20. Given the function: $f(x)=\begin{cases}|18\text{-}x| & \text{if } x<7\\ x-10 & \text{if } x\ge 7\end{cases}$

Evaluate $\lim_{x\to 7} f(x)$

___ 21. Evaluate $\lim_{x\to 3}\frac{x^3-27}{x^2-9}$

Answers:

A. $\frac{9}{2}$

B. -1

C. ∞

D. $\frac{1}{3}$

E. -3

F. 0

G. $\frac{1}{2}$

H. 2

I. $-\frac{1}{2}$

J. $-3, -1, 1$

K. -3

L. $-1, 1, 3$

M. 1

N. $\frac{3}{4}$

O. $2x$

P. $2h$

Q. $2hx$

R. 6

S. Limit Does Not Exist

T. e

U. $\frac{1}{4}$

V. $-\infty$

W. $\frac{1}{2c^3}$

X. $\frac{2}{c^3}$

Y. -5

Z. None of the above