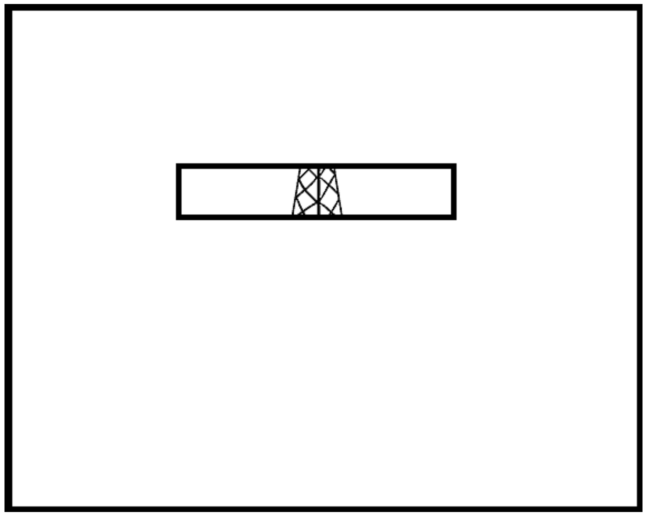
A Turvy for A.P. Calculus Exam

by David Pleacher



A Turvy is a Drawing which has a caption right side up
and has another caption if you turn it topsy-turvy.

<u>Caption for the picture:</u>

"9 2 3 3 9 12   10 15 4 9 11   1 17   17 9 9 18   5 14

7 13 1 11 16   2 18   1 11 19 15 11 9 16   10 11 13 21 22."

<u>Caption for the picture turned upside down:</u>

"9 2 3 3 9 12   10 15 4 9 11   1 17   17 9 9 18   5 14

7 13 1 11 16   2 18   1 11 19 15 11 9 16   10 11 13 21 22

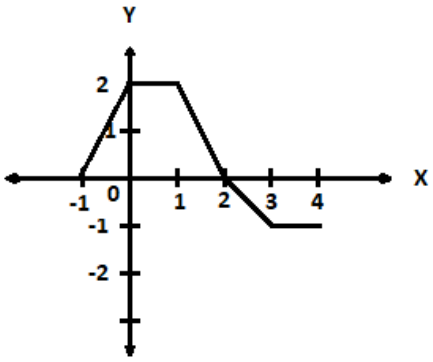
4 8 2 21 8   8 1 17   20 13 17 10   5 9 9 18

15 6 9 11 10 13 11 18 9 16   5 14   1   7 1 18 7   15 3

10 8 2 9 6 9 17."

To determine the titles to this turvy, which was created by Roger Price and published in his book called *Droodles*, solve the 22 A.P. Calculus problems (from the 1998 AB Exam).
Then find the answers to each problem from the choices below.
Replace each numbered blank with the letter corresponding to the answer for that problem.
A calculator should not be used on questions 1 – 21. A calculator is permitted on question #22.

___ 1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?



___ 2. The graph of a piecewise-linear function *f*, for $-1 \le x \le 4$, is shown above.
What is the value of $\int_{-1}^{4} f(x)\,dx$?

___ 3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

___ 4. $\int_{0}^{x} \sin t\,dt =$

___ 5. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

___ 6. $\int_{1}^{e} \left(\frac{x^2 - 1}{x} \right) dx =$

___ 7. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

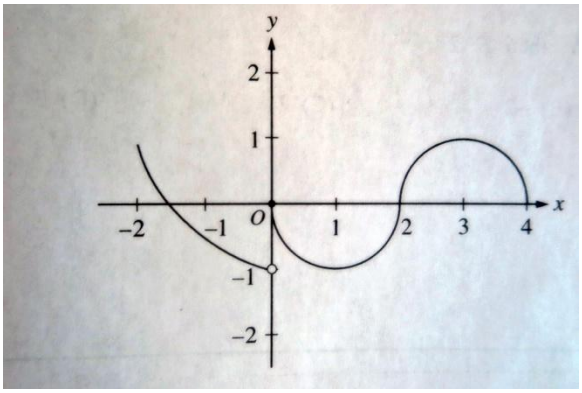
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

___ 8. What is the instantaneous rate of change at x = 2 of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

___ 9. If f is a linear function and 0 < a < b, then $\int_a^b f''(x)\,dx =$

___ 10. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is



___ 11. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, -1) and (3, 1). For what values of x, -2 < x < 4, is f not differentiable?

___ 12. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

___ 13. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

___ 14. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is

___ 15. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when x =

___ 16. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

___ 17. The function f is given by $f(x) = x^4 + x^2 - 2$. On what interval is f increasing?

___ 18. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

___ 19. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $X = 0$ to $x = 2$?

___ 20. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0, 2] ?

___ 21. If $f(x) = \cot 2x$, then $f'\left(\frac{\pi}{6}\right) =$

___ 22. Let *F(x)* be an antiderivative of $\frac{(\ln x)^3}{x}$. If *F(1)* = 0, then *F(9)* =

The turvy used in this puzzle was drawn by Roger Price and appeared in his book called *Droodles.*

Answers: (units have been omitted)

A. -5

B. -3.5

C. $\frac{-8}{3}$

D. -3

E. 0

F. 0.5

G. 1

H. 2

I. 2.5

J. $\frac{26}{9}$

K. 5.827

L. 3

M. $\frac{14}{3}$

N. 21

O. -1 and 0 only

P. 0, 1, and 3 only

Q. -1, 0, and 2 only

R. 0 and 2 only

S. $(0,\infty)$

T. Nonexistent

U. $-e^{-x}\cos\left(e^{-x}\right)$

V. $\frac{e^2}{2}-\frac{3}{2}$

W. $1-\cos x$

X. $1+\cos x$

Y. y = x + 1

Z. None of the above