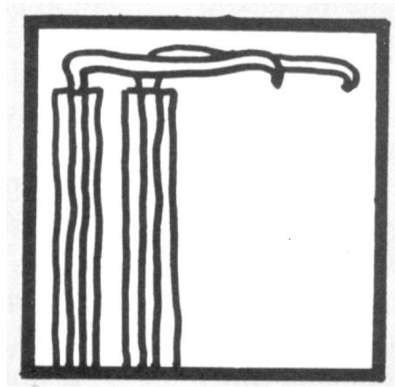
Turvy with Applications of the Integral -- A Puzzle by David Pleacher



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title." In 1985, *Games* Magazine took the Droodle one step further and created the Turvy. Turvies have one explanation right-side-up and an entirely different one turned topsy-turvy. The Turvy above was created by Cynthia Gagnon of South Berwick, Maine and published in *Games* Magazine in May 1986.

Here is the title right-side-up:

"— — — — — — — — — — — — — — — — — — — — — —."

11 12 6   10 15 14 1 8 17 16   7 14   15   9 5 13 13 7 10 15 14 17

Here is the title upside-down:

"— — — — — — — — — — — — — — — — — — — — — — —."

5 14 10 8 17   16 15 4   12 17 15 13 7 14 3   17 8 2   16 9 6 17 16

To determine the titles to this turvy, solve the 17 integral application problems on the next three pages. Then replace each numbered blank with the letter corresponding to the answer for that problem.

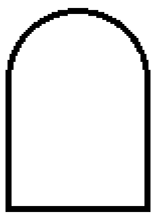
Integral Application Problems:

1. Find the area in square units bounded by the curves $y = x^3 - 2x^2$ and $y = 2x^2 - x^3$.

2. Using your calculator, determine the area of a region bounded by the curves $y = \sin x$, $y = 3x$, and $y = 30 - 3x$.

3. Determine the area of the region bounded by $x = (y-2)^2$ and $y = 4 - x$.

4. The figure below is a square of base 4 meters topped by a semicircle. What is the average height of this figure?



5. Determine the area bounded by $x = 2y^2 - 5$ and $x = y^2 + 4$.

6. Determine the area bounded by $y = x$, $y = -\dfrac{x}{2}$ and $y = 5$.

7. Determine the area of the region bounded by $y = \sin x$, $y = \csc^2 x$, $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$.

8. Determine the area of the region IN THE FIRST QUADRANT bounded by the curves by $y = \sin x \cos^2 x$, $y = 2x\cos(x^2)$ and $y = 4 - 4x$.

Answers:

A. $\dfrac{896\pi}{5}$

B. 73.428

C. 2π

D. $\dfrac{8}{3}$

E. $\dfrac{16\pi}{5}$

F. 73.228

G. $\dfrac{9}{2}$

H. 13

I. $2 - \sqrt{2}$

Integral Application Problems: | Answers:

9.	Determine the number a so that $\int_2^5 x^2\,dx$ is the same as $\int_2^5 a\,dx.$	J.	179.2
10.	A solid is formed by revolving around the x-axis the region bounded by the x-axis and the curve $y=\sqrt{\sin x}$ for $0 \le x \le \pi.$ Determine the volume of the solid.	K.	$\pi + 2$
11.	The acceleration function (in meters per second) and initial velocity are given for an object moving along a straight line: $a(t) = 4t - 1, \quad v(0) = -6.$ Determine the total distance traveled by the object in the first 5 seconds.	L.	0.379
12.	Determine the volume of the solid that results when the region between the curve $y = x$ and the x-axis, from $x = 0$ to $x = 1,$ is revolved around the x-axis.	M.	$\frac{\pi}{2} + 4$
13.	Determine the volume of the solid that results when the region bounded by $y = x$ and $y = x^2$, from $x = 0$ to $x = 1,$ is revolved about the x-axis.	N.	$\frac{2\pi}{35}$
14.	Determine the volume of the solid that results when the region bounded by $x = y^2$ and $x = y^3$, from $y = 0$ to $y = 1,$ is revolved about the y-axis.	O.	$\frac{75}{2}$
15.	Determine the volume of the solid that results when the region bounded by $y = x^2$ and $y = 4x,$ is revolved about the line $y = -2.$	P.	$\frac{\pi}{6}$

Integral Application Problems: Answers:

16. $\begin{cases} \text{Determine the volume of the solid that results when} \\ \text{the region bounded by } y = 2\sqrt{x},\ \ x = 4 \ \text{ and } y = 0, \text{ is} \\ \text{revolved around the y-axis (use cylindrical shells).} \end{cases}$

17. $\begin{cases} \text{Determine the volume of the solid that results when} \\ \text{the region bounded by } y = x^3,\ \ x = 2 \ \text{ and the x-axis,} \\ \text{is revolved around the line } \ x = 2. \end{cases}$

Q. $\dfrac{64\pi}{5}$

R. $\dfrac{2\pi}{15}$

S. $\dfrac{256\pi}{5}$

T. $\dfrac{349}{6}$

U. 36

V. 18

W. $\dfrac{\pi}{3}$

X. 64

Y. $2+\sqrt{2}$

Z. 0

Units were omitted in the answers on purpose because some of the answers might have been too obvious.