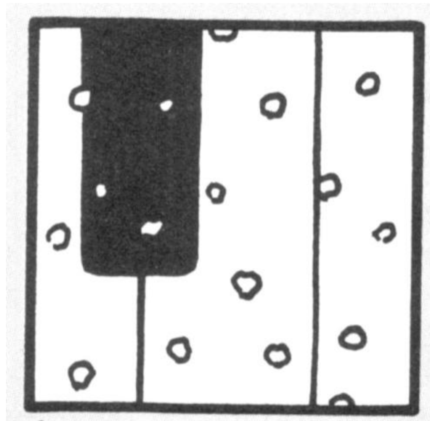
Turvy for Integration -- A Puzzle by David Pleacher



Back in 1953, Roger Price invented a minor art form called the Droodle, which he described as "a borkley-looking sort of drawing that doesn't make any sense until you know the correct title." In 1985, *Games* Magazine took the Droodle one step further and created the Turvy. Turvies have one explanation right-side-up and an entirely different one turned topsy-turvy. The Turvy above was created by Dan White of Helena, Montana and published in *Games* Magazine in May 1986.

Here is the title right-side-up:

"— — — — — —   — — — — — —   — —   —   — — — — —."

9 18 18 2 12 19   9 3 7 11 8 17   18 16   10   4 12 10 16 18

Here is the title upside-down:

"— — —   — — — — — — —   — — — — — — —   — —   —   — — — —

10 8 19   13 12 16 9 18 13 16   1 10 13 2 12 16 6   8 5   10   15 10 13 13

— — — — —   — —   —   — — — — — — — — —."

14 19 16 9 19   12 16   10   17 16 18 1 17 15 18 3 11

To determine the titles to this turvy, solve the 19 Integral problems on the next two pages. Then replace each numbered blank with the letter corresponding to the answer for that problem.

Basic Integration Problems:

1. Evaluate $\int\left(\frac{1}{x^4}+\frac{1}{x^2}+x^{10}\right)dx$

2. $\int 4\left(\frac{1}{x}+x^{\frac{2}{5}}\right)^2 dx=$

3. $\int \frac{x^2-9}{x+3}dx=$

4. $\int\left(\sin x-3\cot x\sin x\right)dx=$

5. $\int\left(2x^{-\frac{3}{7}}+\frac{5}{\sin^2 x}\right)dx=$

6. $\int \csc^2 x\cos x\,dx=$

7. $\int \frac{d}{dx}\left(3x^{-2}+\tan x-4\right)dx=$

8. $\int \cos(2x)\sqrt{\sin(2x)}\,dx=$

9. $\int \frac{x}{\sqrt{1-x^2}}dx=$

10. Given that $g'(x)=\left(\sin x\right)\left(5+5\cos x\right)^3$, find $g(x)$ if $g(0)=0$

11. $\int 8x\left(x^2+7\right)^3 dx=$

12. $\int \frac{x^2}{\left(2x^3-12\right)^4}dx=$

Answers:

A. $-\frac{1}{20}\left(5+5\cos x\right)^4+500$

B. $\frac{1}{3}\sin^{\frac{3}{2}}\left(2x\right)+C$

C. $-\sqrt{1-x^2}+C$

D. $\left(5+5\cos x\right)^4+500$

E. $\frac{1}{4}\tan^2\left(3x^2\right)+C$

F. $-\left(3-\frac{1}{x}\right)^{-1}+K$

G. $-\csc x+C$

H. $-\frac{1}{6}\cos^{-1}\left(3x^2\right)+C$

I. $\frac{-1}{18\left(2x^3-12\right)^3}+C$

J. $-\sin\left(\sin x\right)+C$

K. $-\frac{4}{x}+20x^{\frac{2}{5}}+\frac{20}{9}x^{\frac{9}{5}}+K$

L. $\frac{2}{7}\left(2+x^{\frac{1}{2}}\right)^7+C$

Basic Integration Problems:

13. $\int \frac{\left(2+\sqrt{x}\right)^6}{\sqrt{x}}\,dx =$

14. $\int \left(3-\frac{1}{x}\right)^{-2}\left(\frac{1}{x^2}\right)dx =$

15. $\int \frac{\cos x}{\left(2-3\sin x\right)^4}\,dx =$

16. $\int \frac{x}{\cos^2\left(3x^2\right)}\,dx =$

17. $\int \frac{4x^3-3}{\left(x^4-3x\right)^2}\,dx =$

18. $\int \cos x \cos\left(\sin x\right)\,dx =$

19. $\int 3x\tan\left(3x^2\right)\sec^2\left(3x^2\right)\,dx =$

Answers:

M. $\left(x^2+7\right)^4+C$

N. $\frac{1}{6}\tan\left(3x^2\right)+C$

O. $\sin\left(\sin x\right)+C$

P. $-\cos x-3\sin x+C$

Q. $\frac{1}{2}\tan^2\left(3x^2\right)+C$

R. $\frac{x^2}{2}-3x+C$

S. $\frac{-1}{\left(x^4-3x\right)}+C$

T. $\frac{1}{9}\left(2-3\sin x\right)^{-3}+C$

U. $3x^{-2}+\tan x+K$

V. $\frac{1}{7}\left(2+\sqrt{x}\right)^7+K$

W. $-\frac{x^{-3}}{3}-x^{-1}+\frac{x^{11}}{11}+C$

X. $-\cos x+3\sin x+C$

Y. $\frac{7}{2}x^{\frac{4}{7}}-5\cot x+C$

Z. $3x^2+\tan x+C$