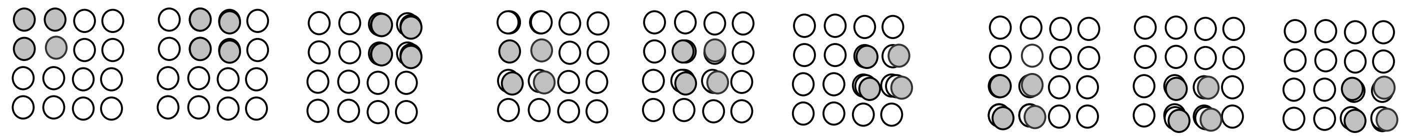
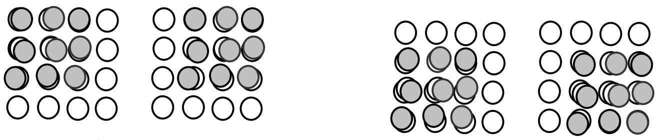
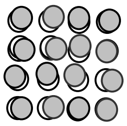
2X2 rectangles





3X3 rectangles



4X4 rectangles

From the above I can try and generalize a formula for the number of squares that can be produced from any square matrix of circles of dimension n x n. The first thing I notice is that the smallest square must have two circles on a side, hence a 2 x 2, the largest square, an n by n.

Now the number of 2 x 2s across each row is n –1. The number of rows that can be counted is also n –1. In other words the number of 2 x 2s = (n – 1)(n – 1)

The number of 3 x 3s across each row is (n – 2). The number of rows that can be counted is (n – 2).
Total number of 3 x 3s = (n – 2)(n – 2).

Continue this until you get the final square which whose size is n x n and can be found by multiplying (n – (n-1))(n – (n-1)) = 1

Now I have to add up the number of squares from a 2x2, 3x3, …, nxn to get the final answer. So this is looking like a summation problem.

Making a table:

k	Dimension of circle matrix	Sequence to sum	Sum
1	1x1	0	0
2	2x2	1	1
3	3x3	4 + 1	5
4	4x4	9 + 4 + 1	14
5	5x5	16 + 9 + 4 + 1	20
M	mxm	$(m-1)^2$	

The last column from the bottom row is the sum of $(m – 1)^2$ from $\sum_{1}^{m}(m-1)^2$