Three-Ring Circus Problem

Solution: Let the price of three rings be as follows:

Ruby- x, Diamond-y and Sapphire-z. Then

$$x = 2(y - z), \qquad \qquad \qquad \dots(1)$$

$$y = 4(z - x), \qquad \qquad \qquad \dots(2)$$

And $\quad z = 2000n, n \in N.$ $\qquad \qquad \qquad \dots(3)$

From (1) and (2), we have

$$x = 2y - 4000n. \qquad \qquad \qquad \dots(4)$$

From (2) and (3), we have

$$y = 8000n - 4x. \qquad \qquad \qquad \dots(5)$$

From (4) and (5), we have

$$x = 16000n - 8x - 4000n$$

$$\Rightarrow \quad x = \frac{4000n}{3}. \qquad \qquad \qquad \dots(6)$$

From (5) and (6), we have

$$y = 8000n - \frac{16000n}{3} = \frac{8000n}{3}. \qquad \qquad \qquad \dots(7)$$

From the given condition and (3), (6) and (7), we have

$$20000 < \frac{4000n}{3} + \frac{8000n}{3} + 2000n < 50000$$

$$\Rightarrow \quad 20000 < 6000n < 50000.$$

This inequality is satisfied for $n = 4, 5, 6, 7, 8.$

Therefore $(x, y, z) = \left(\frac{16000}{3}, \frac{32000}{3}, 8000\right), \left(\frac{20000}{3}, \frac{40000}{3}, 10000\right),$ $(8000, 16000, 12000),$ $\left(\frac{28000}{3}, \frac{56000}{3}, 14000\right)$ and $\left(\frac{32000}{3}, \frac{64000}{3}, 16000\right).$ Ans.