

Sum Thing Special Problem

Solution: Let a be the first term of the set of consecutive n integers whose sum is $S = 296$. Then

$$S = \frac{n}{2}(2a + n - 1) = 296$$

$$\Rightarrow n(2a + n - 1) = 592 = 2^4 \times 37. \quad \dots(1)$$

$$\Rightarrow 2a = \frac{592}{n} - n + 1. \quad \dots(2)$$

Since $2a$ is even, $\frac{592}{n} - n + 1$ must be even. Therefore $\frac{592}{n} - n$ must be odd.

From (1), we see that 16, 37 and 592 are the factors of 592.

$$\text{If } n = 16 \text{ then } 2a = \frac{592}{16} - 16 + 1 = 37 - 16 + 1 = 22 \Rightarrow a = 11.$$

Therefore the required set is given by

$$A = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\}.$$

$$\text{If } n = 37 \text{ then } 2a = \frac{592}{37} - 37 + 1 = 16 - 37 + 1 = -20 \Rightarrow a = -10.$$

Therefore the required set is given by

$$B = \{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, \dots, 26\}.$$

$$\text{If } n = 592 \text{ then } 2a = \frac{592}{592} - 592 + 1 = 1 - 592 + 1 = -590 \Rightarrow a = -295.$$

Therefore the required set is given by

$$C = \{-295, -294, \dots, 0, 1, \dots, 295, 296\}.$$

These are the three required sets. There may be some more sets.

From:

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