

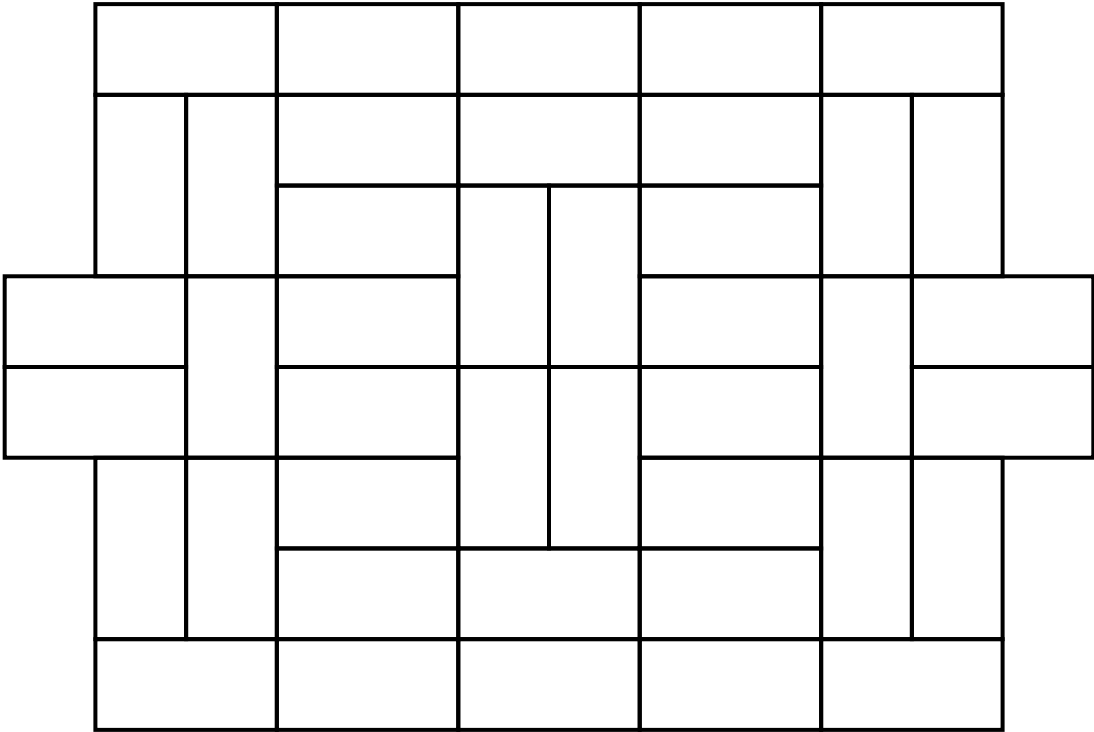
Coloring Dominoes*

If we suppose that any two dominoes touching at either an edge or a corner must have different colors. Then we can show that at least five colors are required.

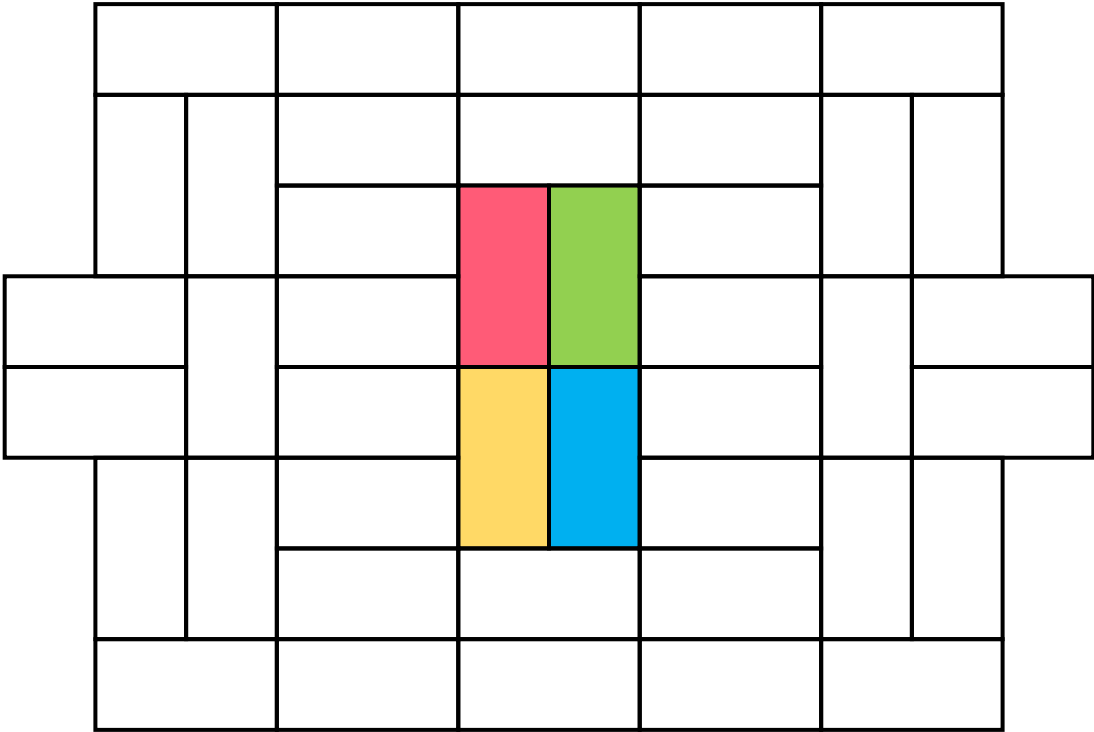
* Or more precisely, a region that is tiled with dominoes.

by Rod Fletcher
2026-06-22

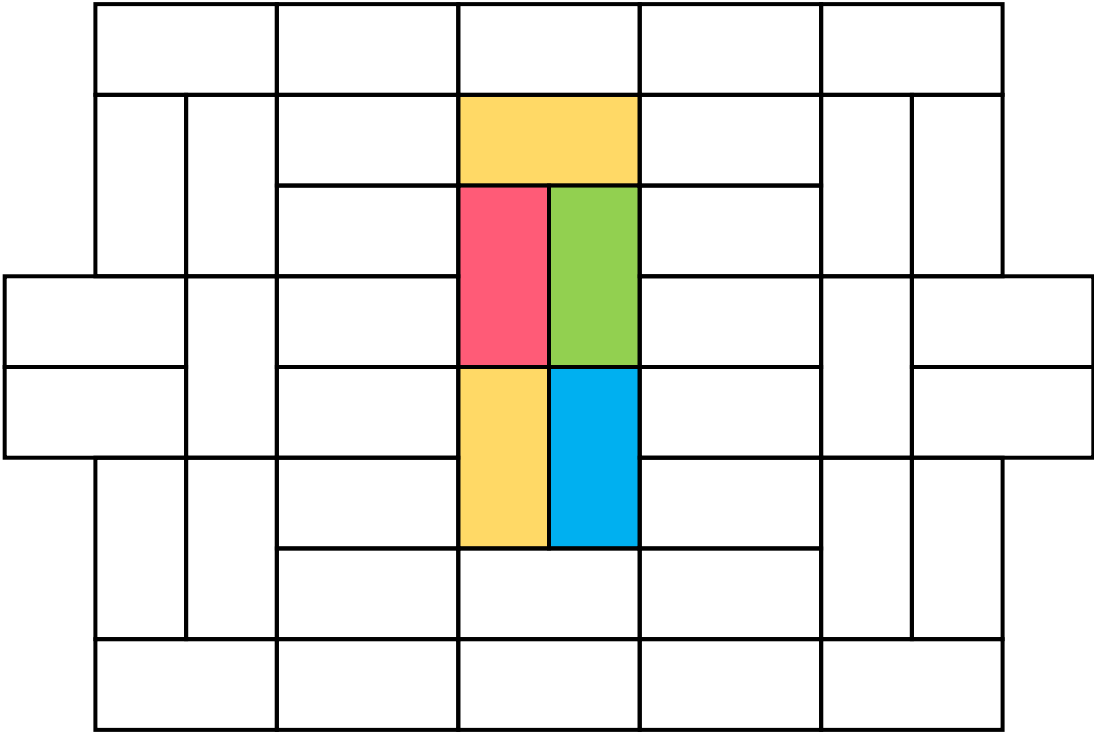
This is a configuration of dominoes that cannot be colored with four colors.



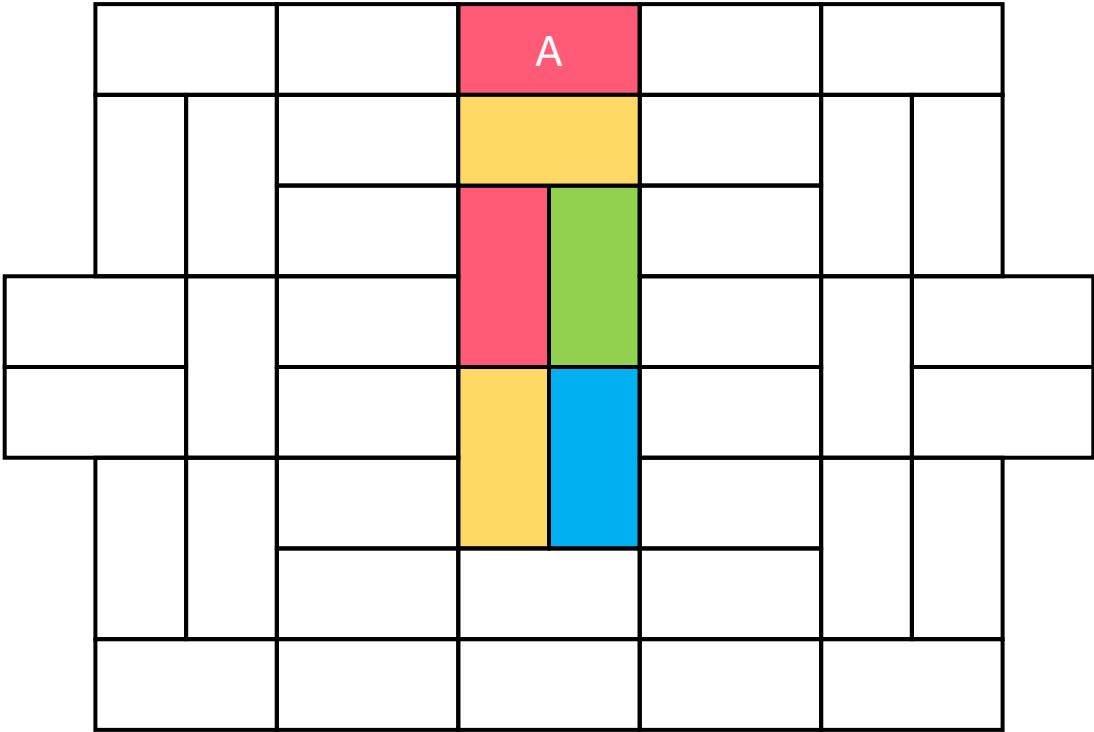
We will prove it by contradiction. We assume that it *can* be colored with 4 colors. The four squares in the center require all 4 colors.



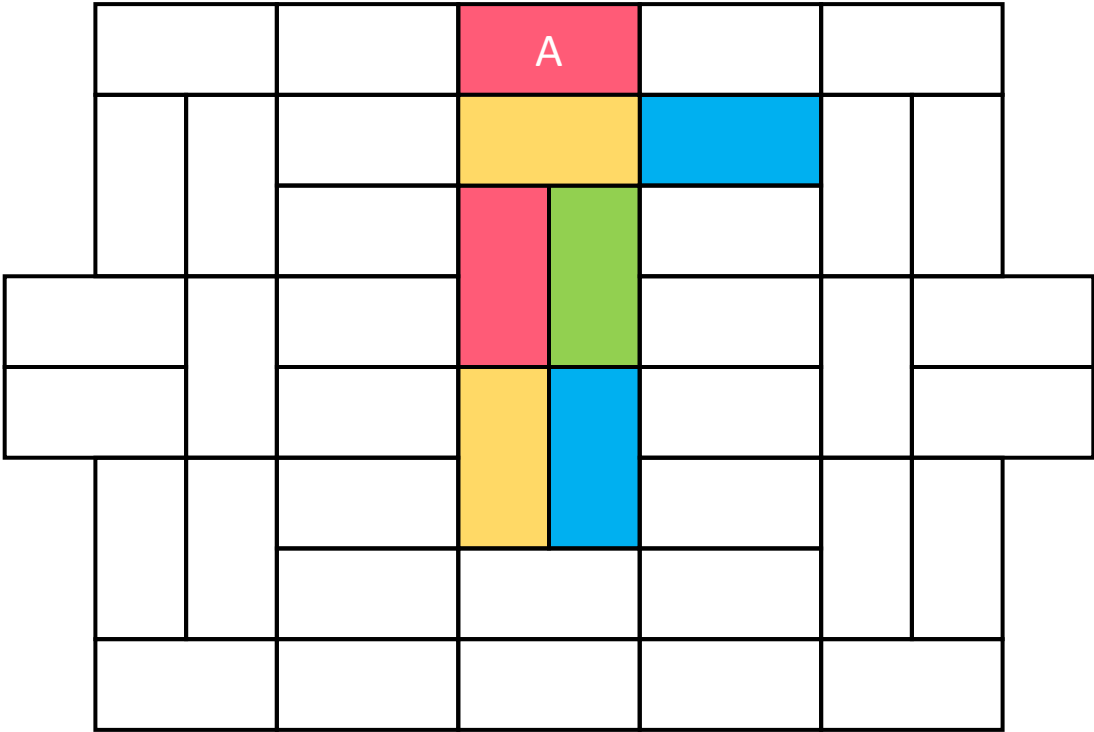
The domino above the four in the center must be either yellow or blue.
Since the entire figure has left-right symmetry, it suffices to try yellow.



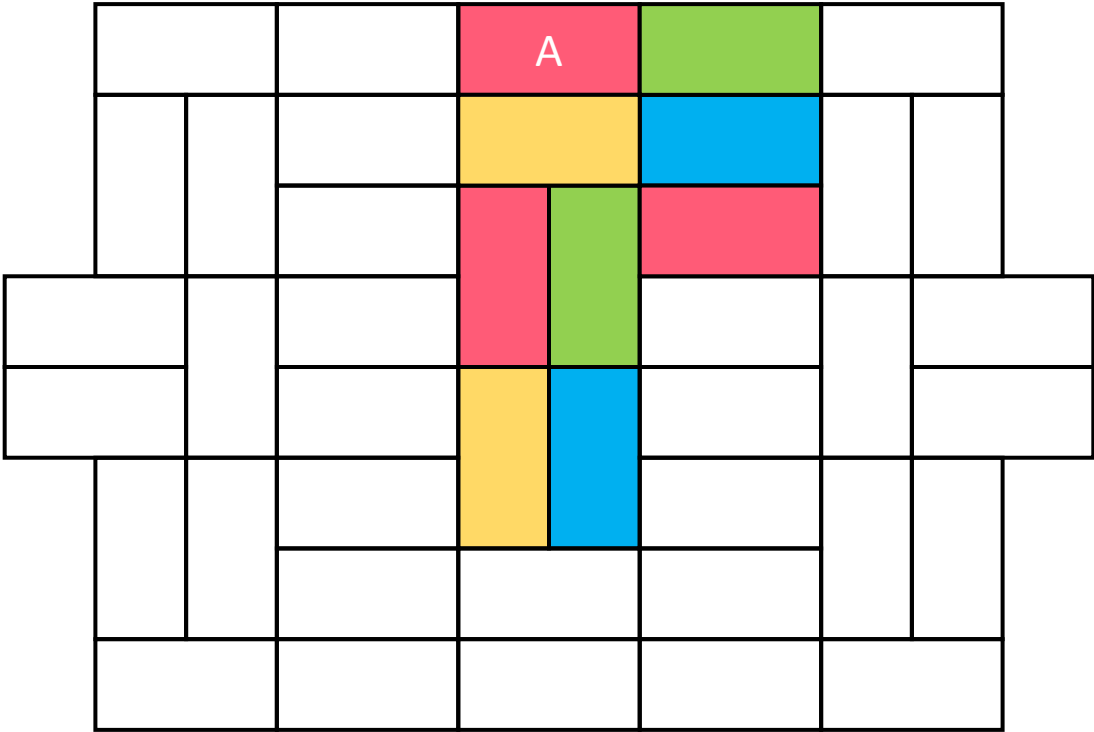
The domino labeled A can be any color other than yellow.
We will try each one in turn. First red.



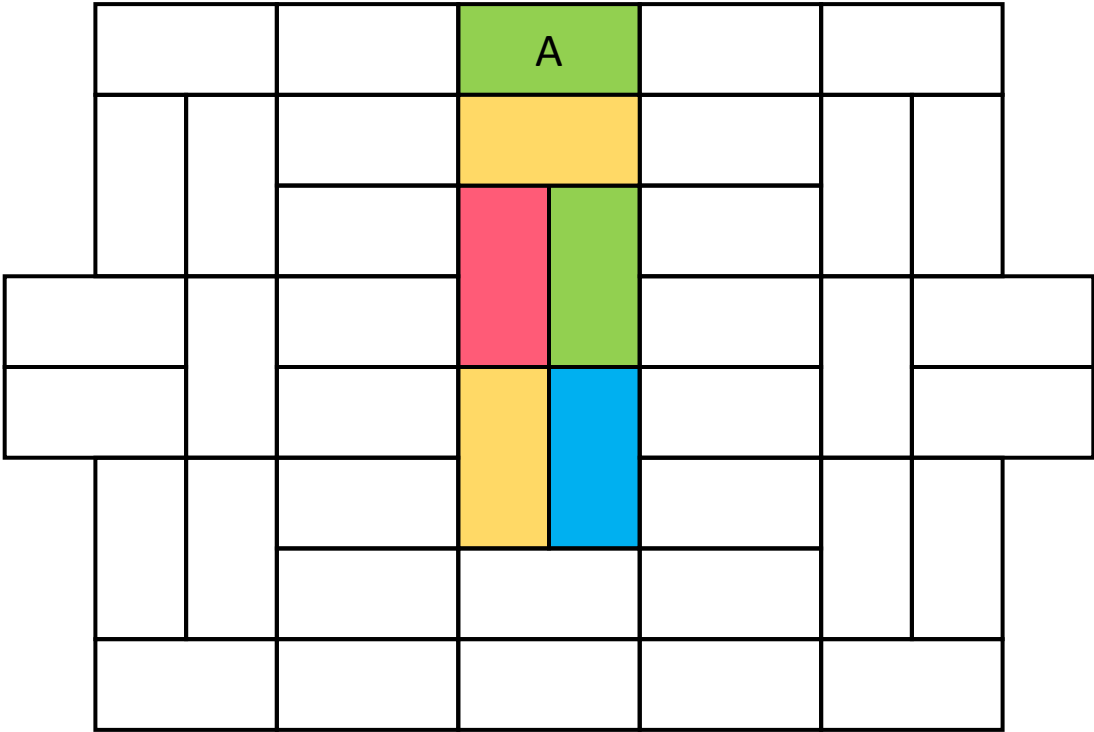
Now there is a domino that is forced to be colored blue.



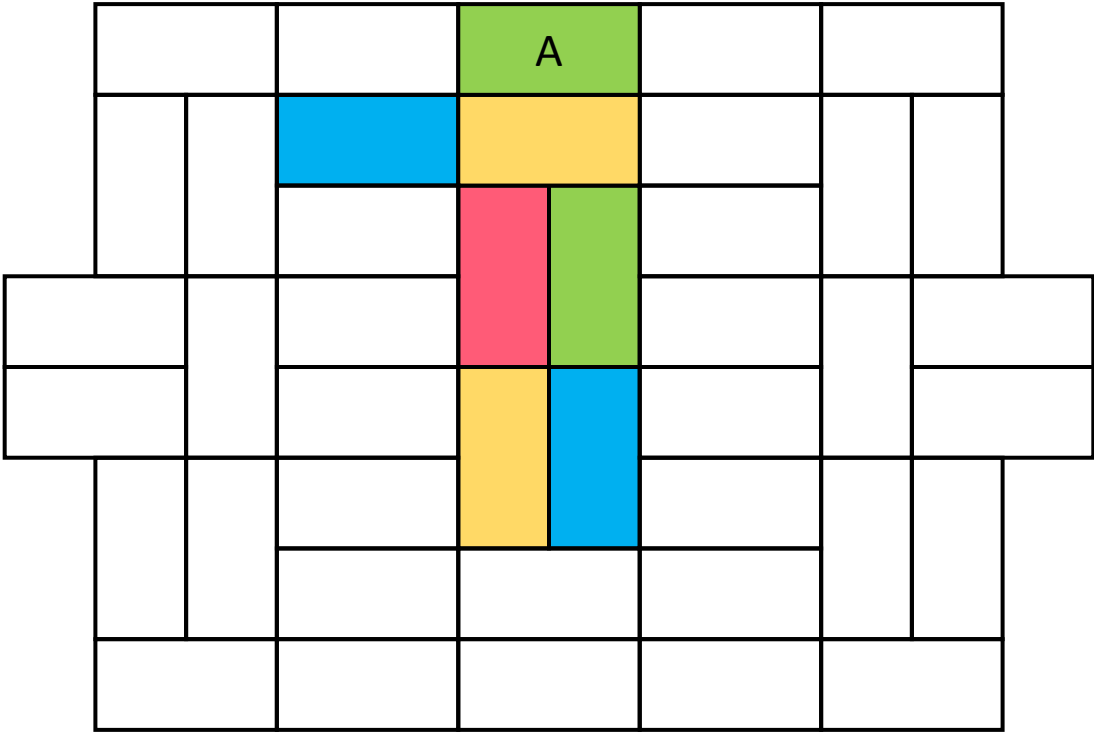
The two dominoes adjacent to the blue one are likewise forced.



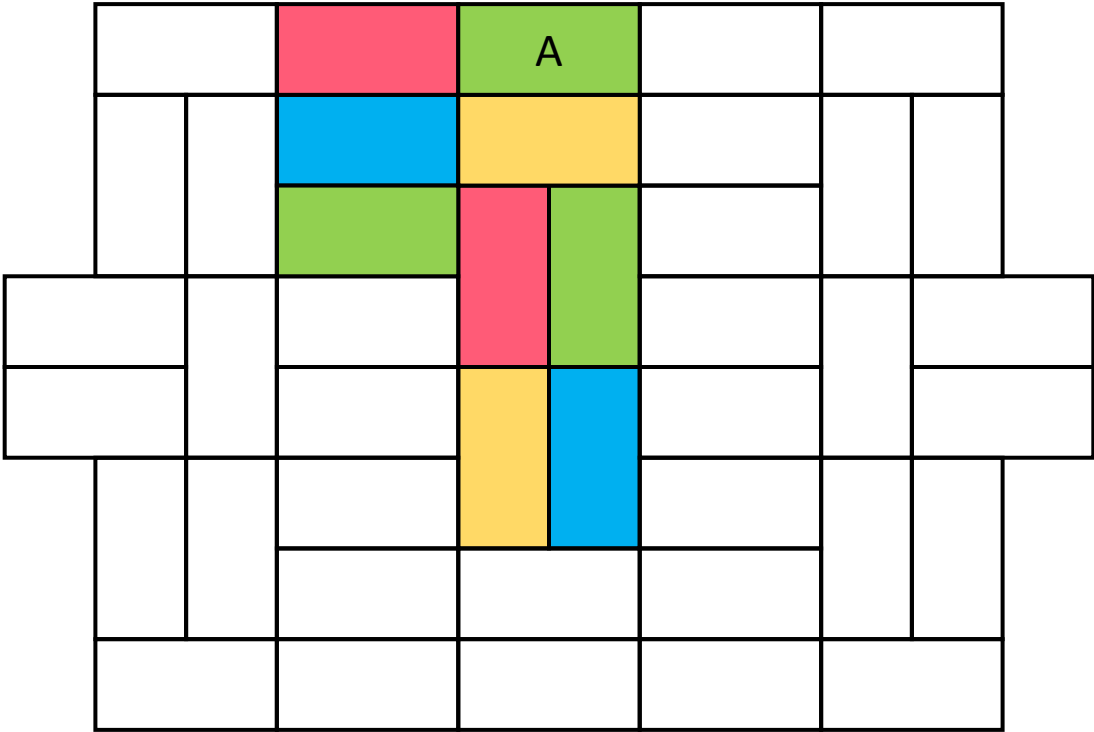
Choosing red for A didn't work. Now we will try green.



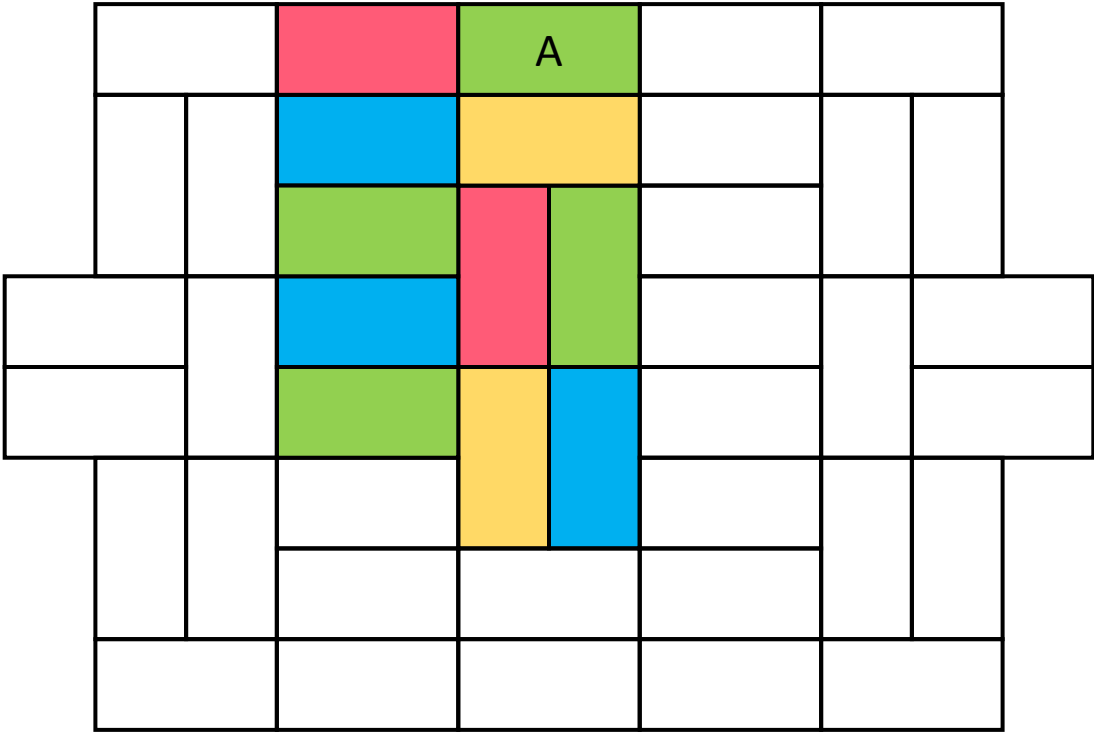
There is one domino that is forced to be colored blue.



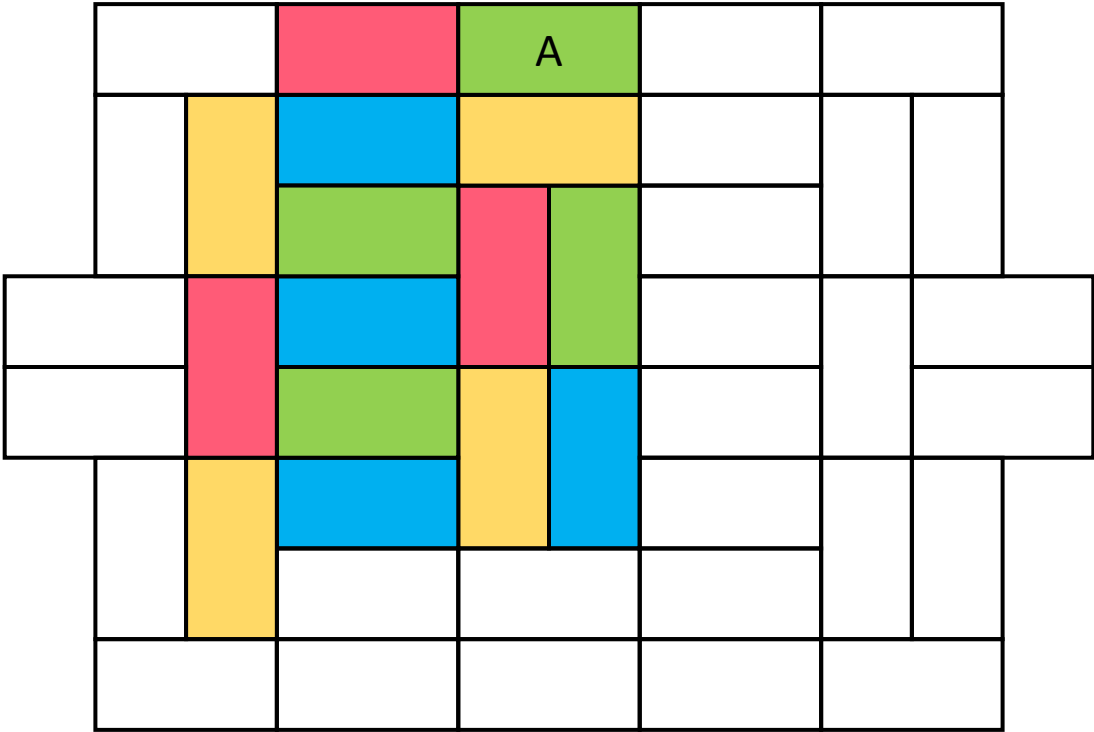
The two dominoes adjacent to the blue one are likewise forced.



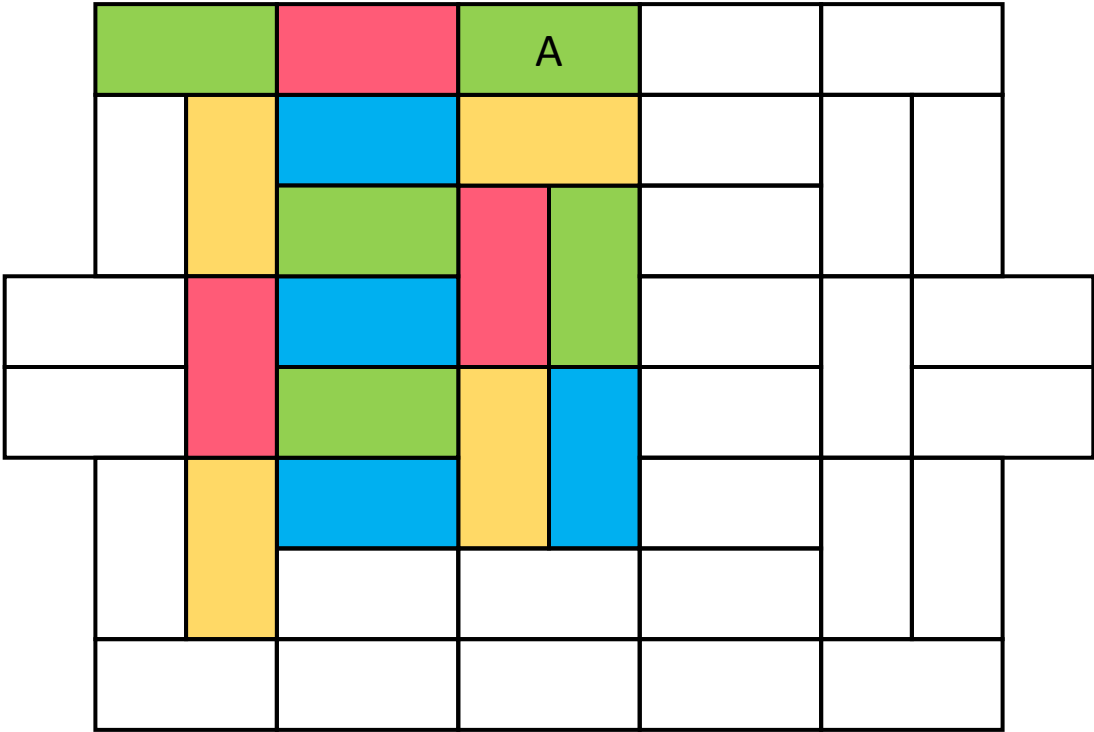
The next two dominoes going down are forced to alternate between blue and green.



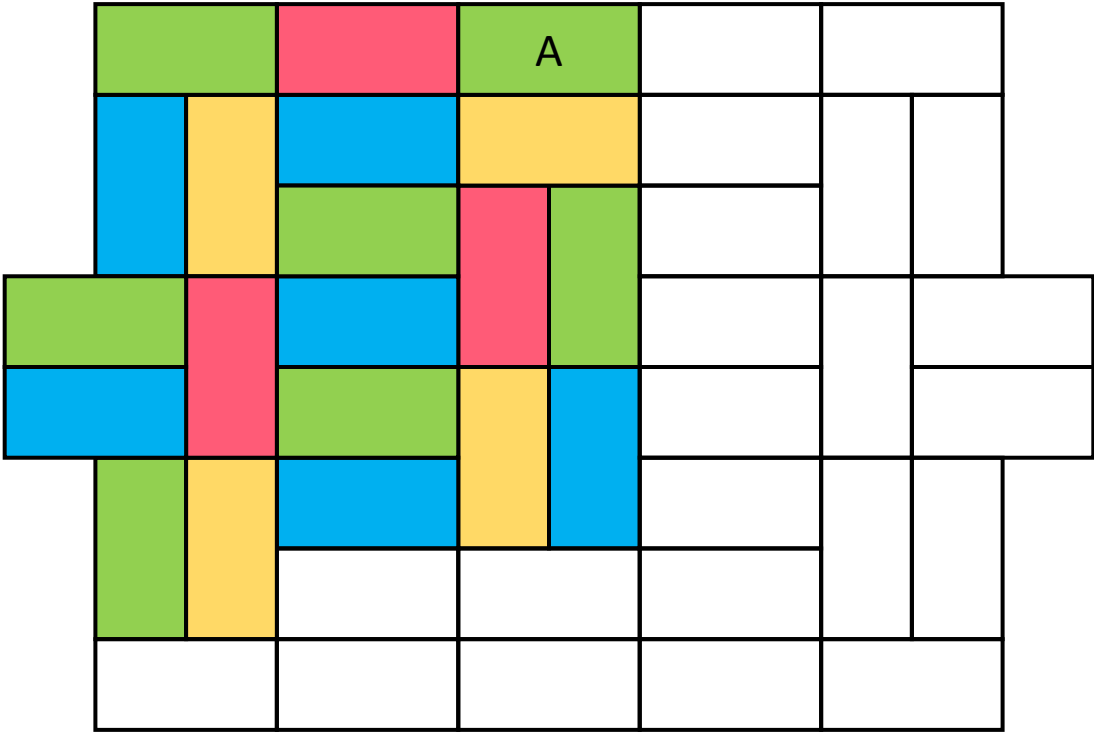
The alternating patterns are forced to continue, first blue, then yellow.



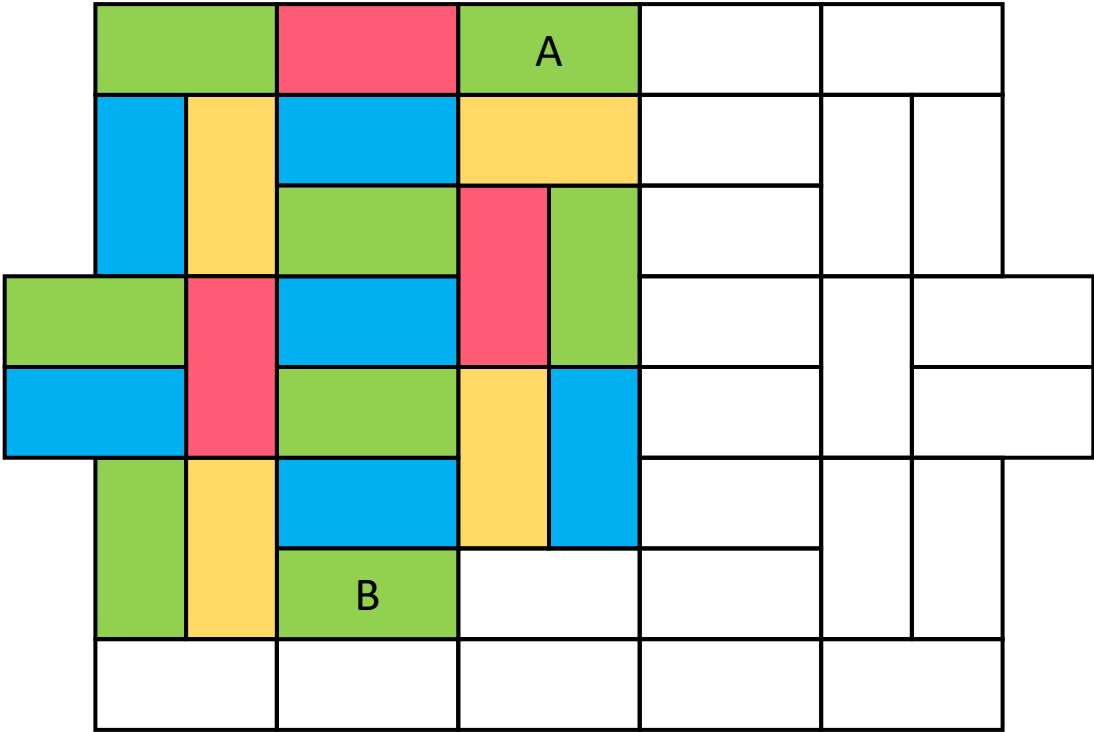
There is still one place where a color is forced.
The upper left domino must be colored green.



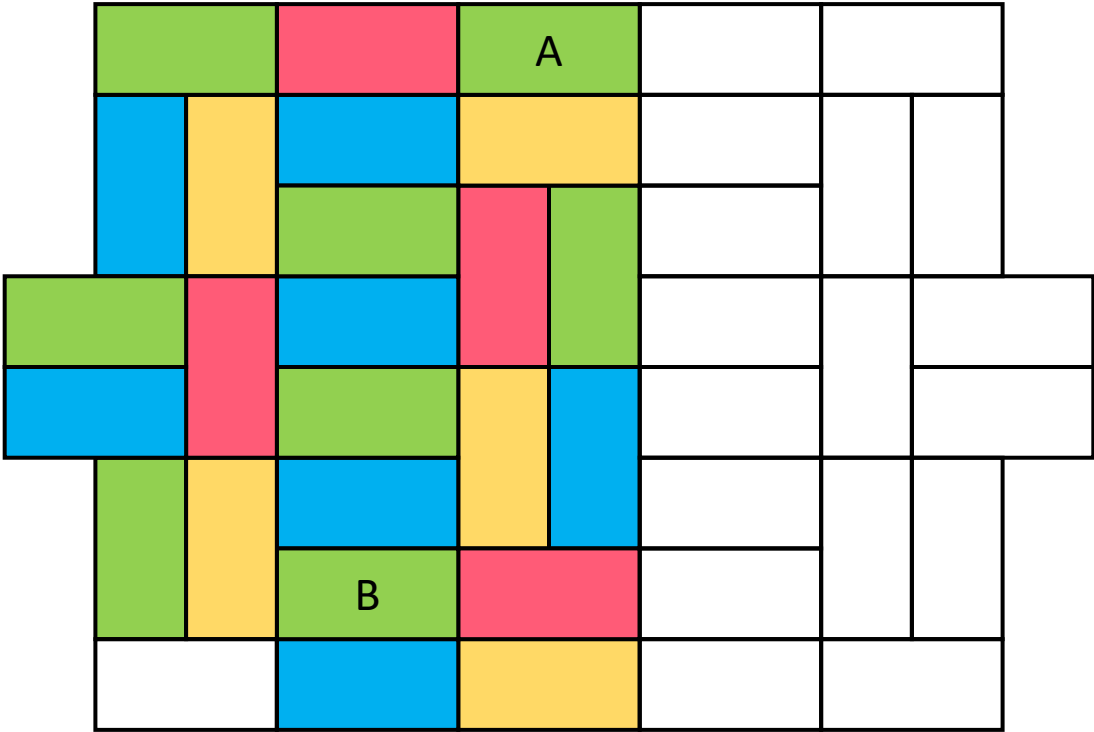
Moving counterclockwise around the left side, the dominoes are forced to alternate between blue and green.



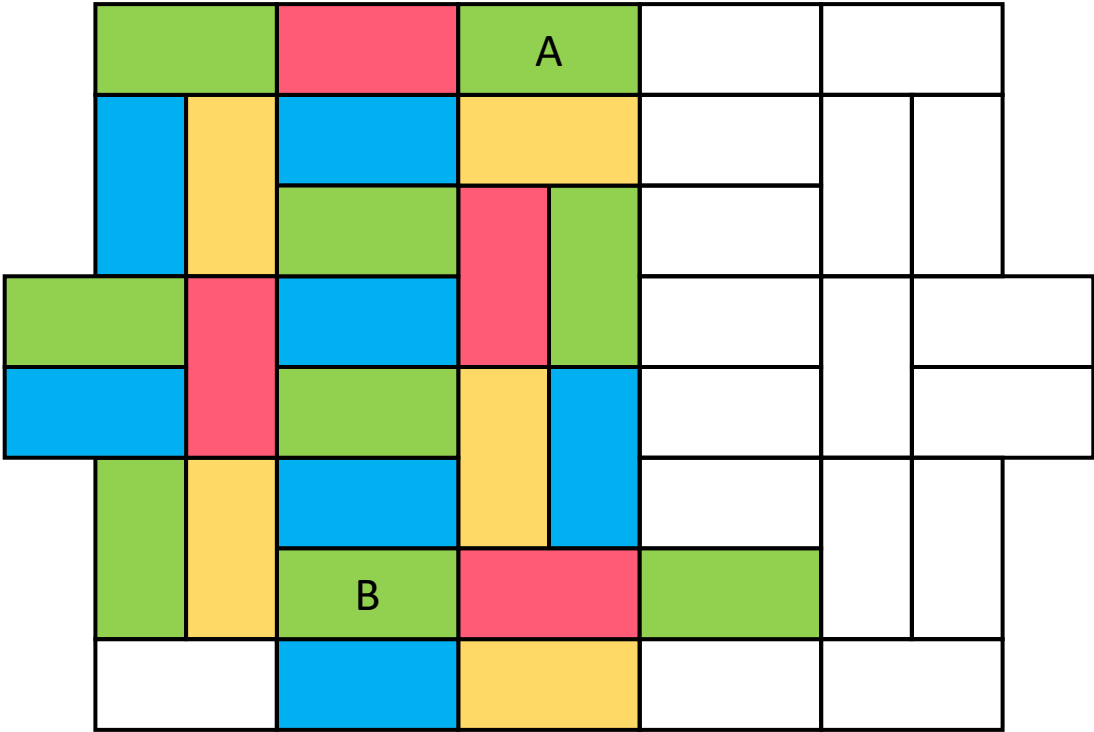
Therefore, we must try a different color for B.
The only color remaining is green.



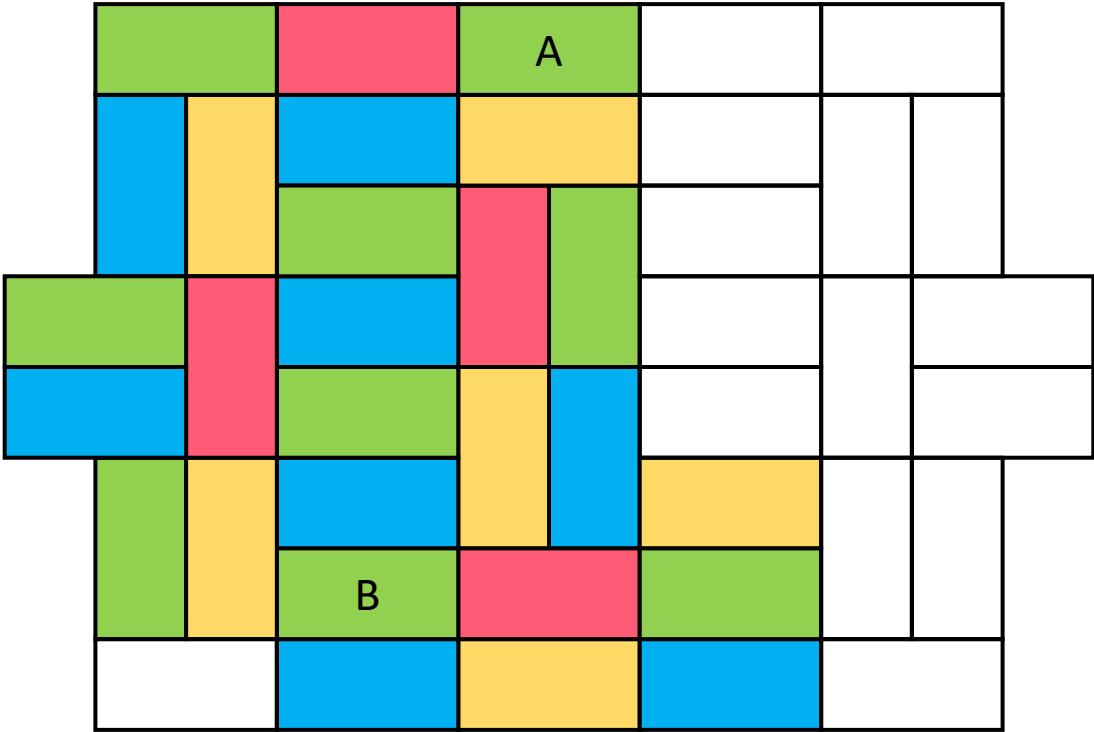
The dominoes below and to the right of B are forced to be red, blue, and yellow, in that order.



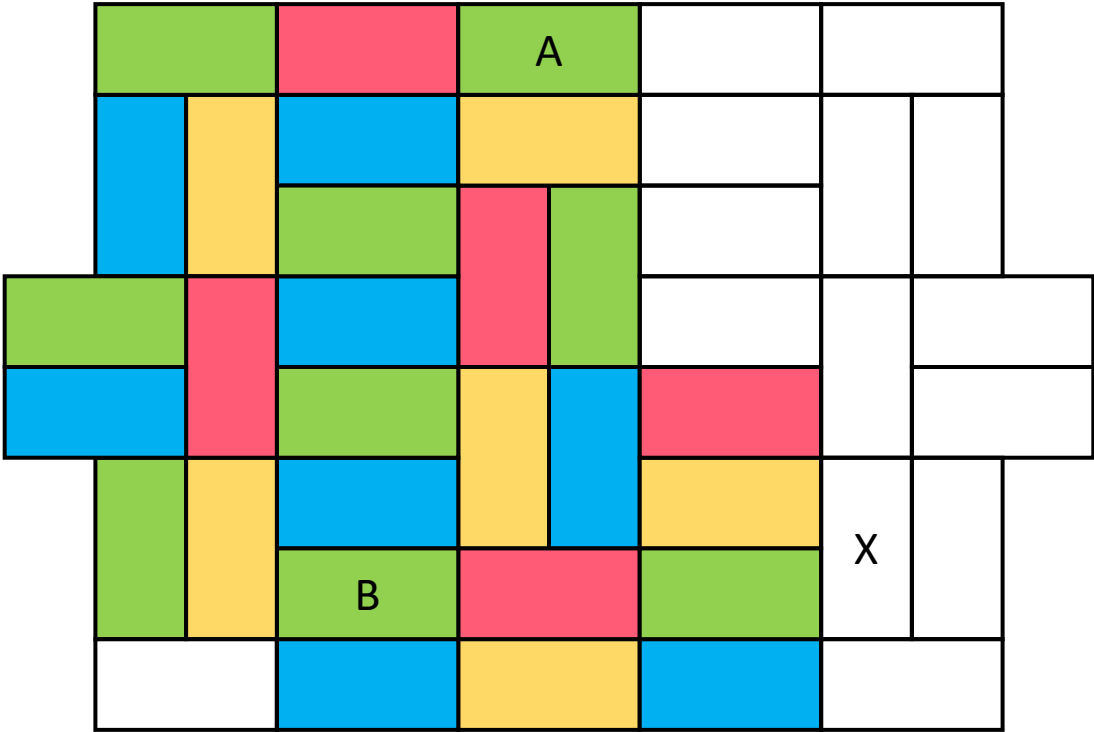
One domino farther to the right is forced to be green.



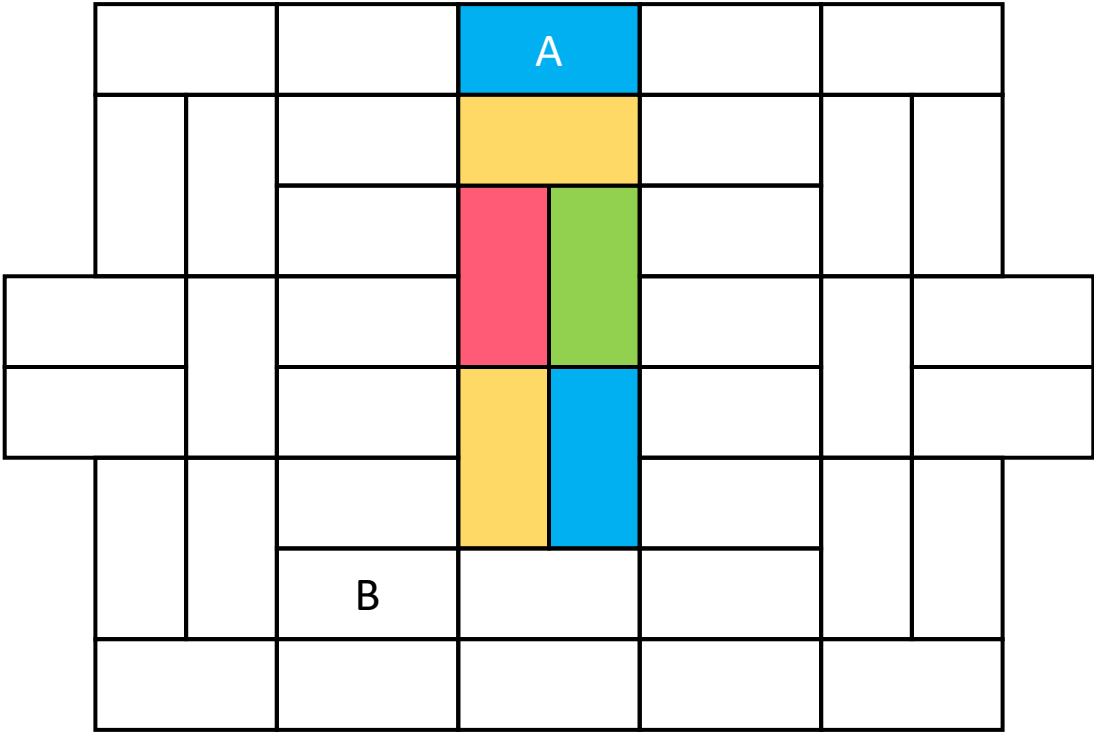
The two dominoes adjacent to the green one are likewise forced.



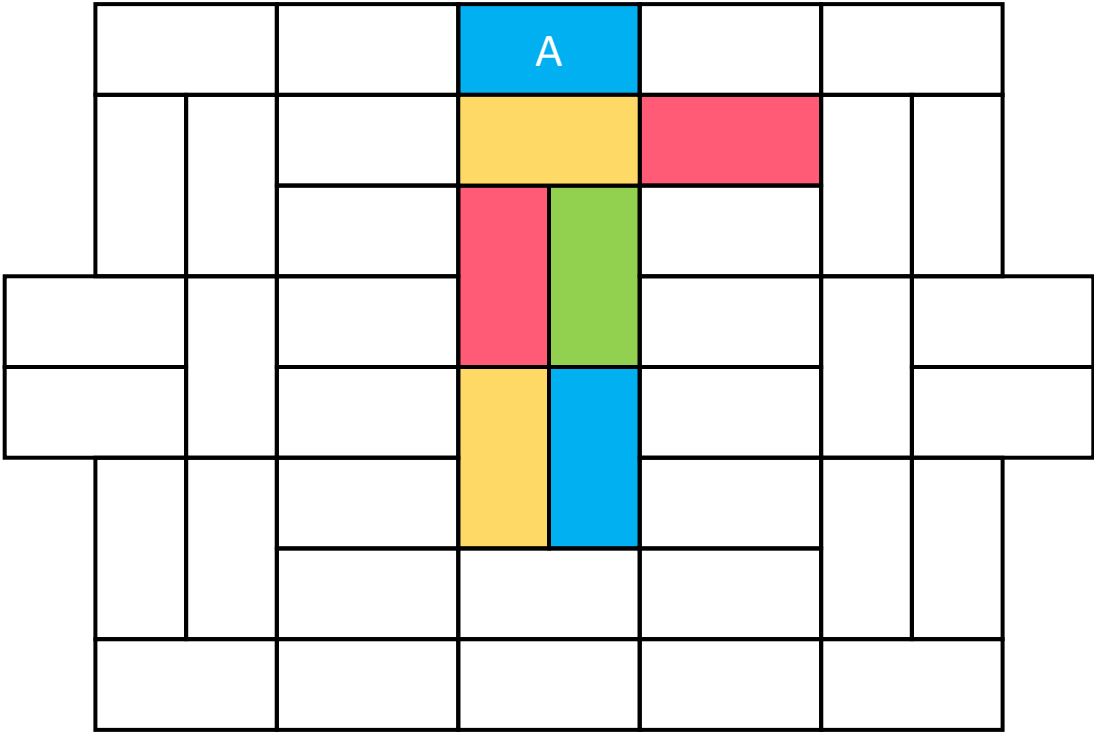
The next domino up is forced to be red, thus forcing us to use a 5th color for the domino labeled X, a contradiction.



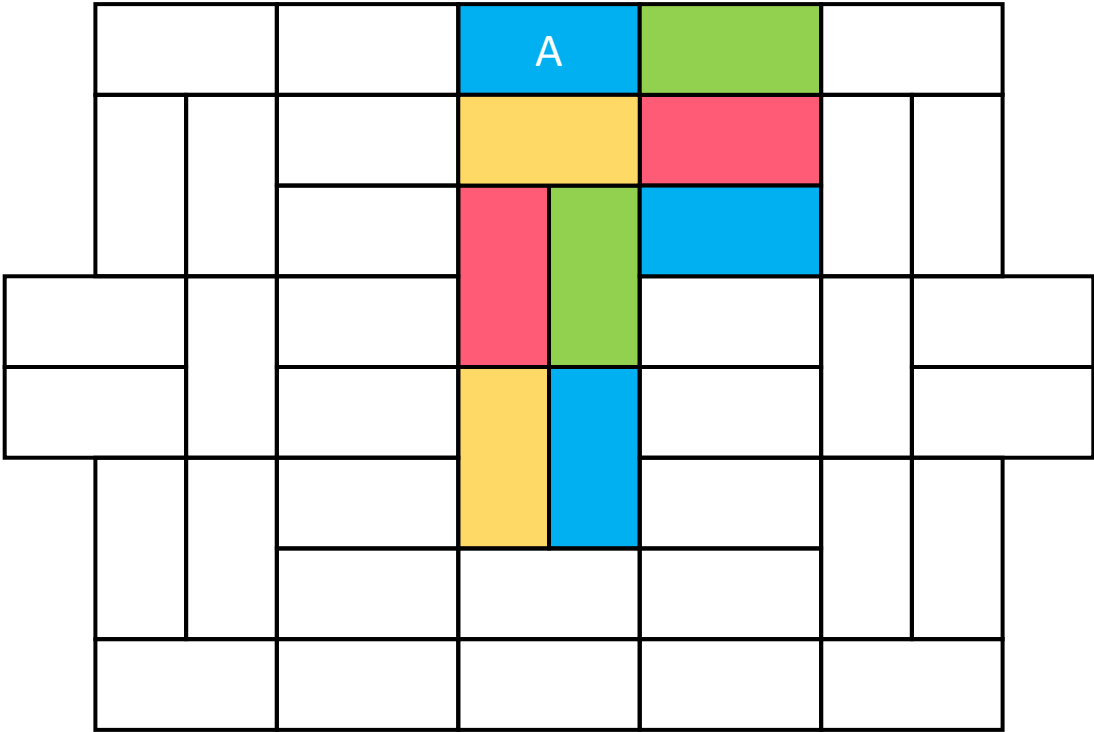
Both choices for B have been exhausted, so now we must go back and try a different color for A. The only one left is blue.



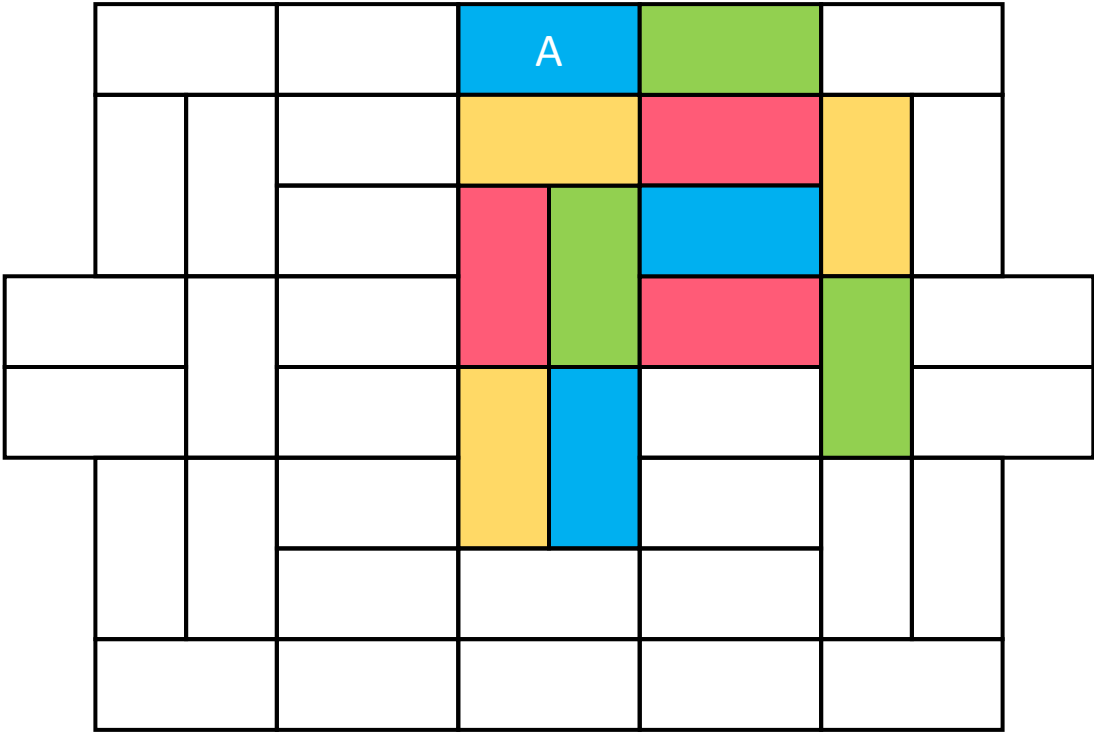
The dominoes on either side of the yellow one are forced to a particular color. The one on the right must be red.



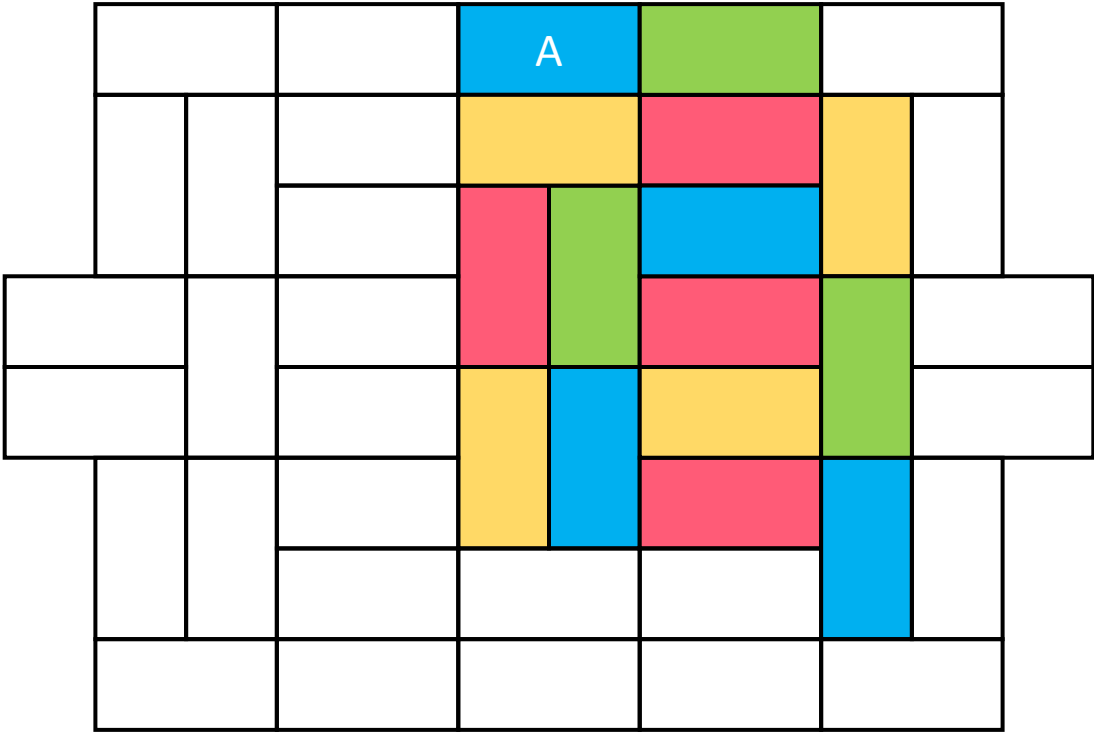
The two dominoes adjacent to the red one are likewise forced.



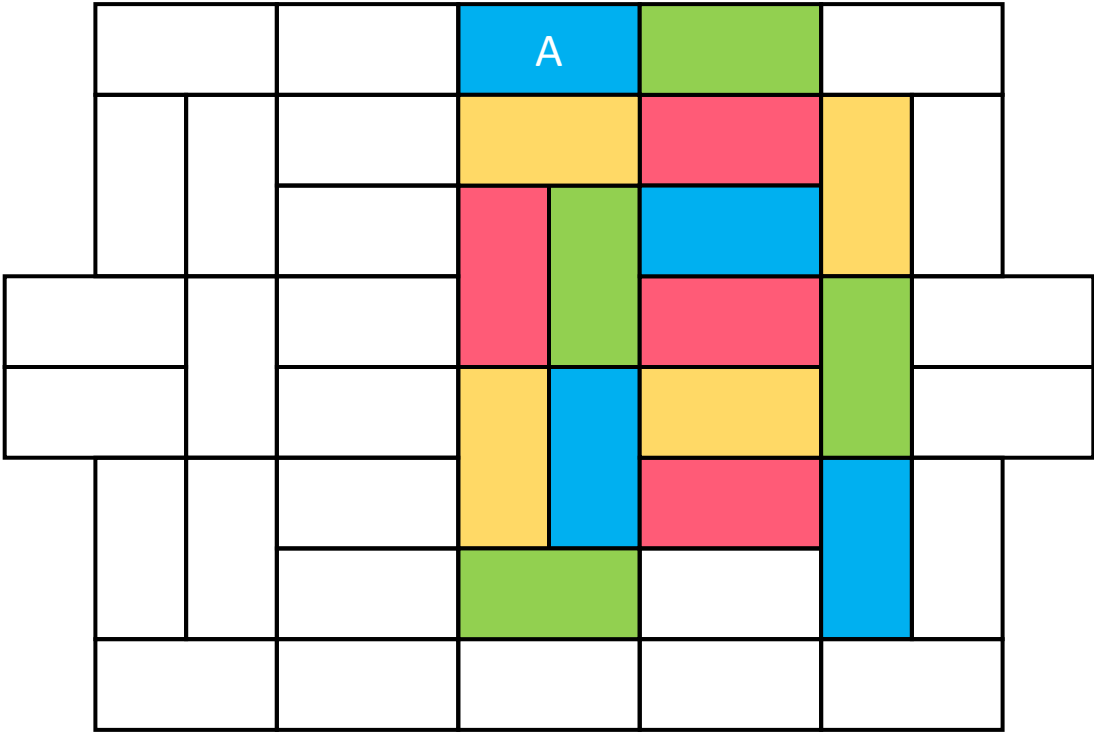
The other dominoes surrounding the blue one just colored are forced to be yellow, red, and green, in that order.



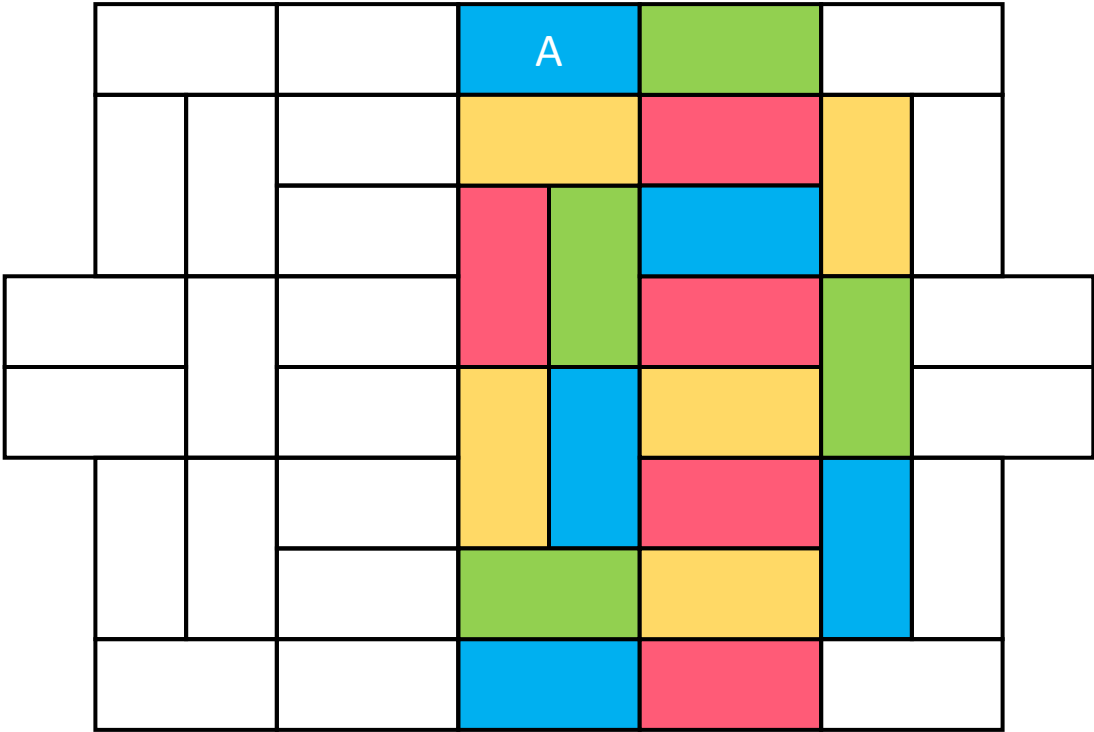
Three more dominoes moving counterclockwise around the green one are forced to be yellow, red, and blue, in that order.



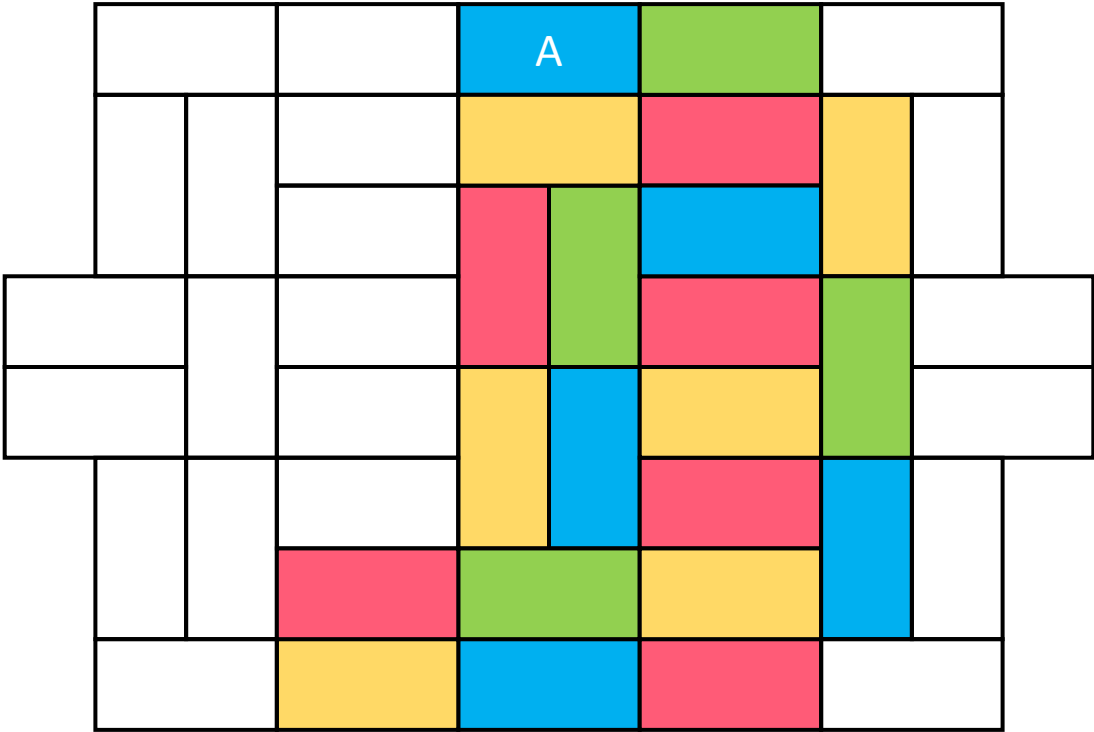
Now the horizontal domino below the four in the middle is forced to be green.



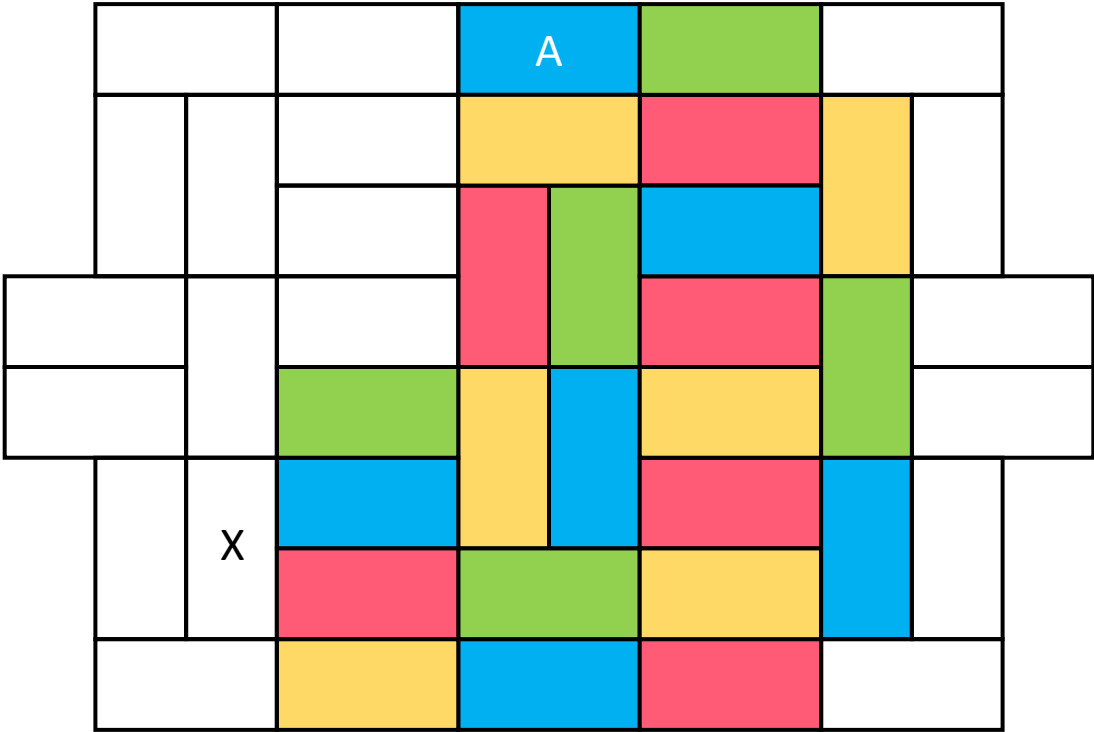
Three more dominoes moving clockwise around the green one just colored are forced to be yellow, red, and blue, in that order.



Two dominoes to the left of the same green one are forced to be red and yellow, in that order.



Two next two dominoes moving up are forced to be blue and green.
The domino labeled X requires a 5th color, contradicting our assumption.



As we have run out of possibilities for A, we may conclude that it is impossible to color this figure using only 4 colors.

